Optimizing Control of Chemical Plants under Uncertain Parameters: a Multiobjective Optimization Approach*

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Abstract. This paragraph shall summarize the contents of the paper in short terms. Xxxxxxxxxx xxxxxx cccccc xxxxxxxx ccc xxxxx.

Keywords: optimal control, nonlinear system, dynamic programming.

1. Introduction

Xxxxxxxxxx (Bellman, 1957) xxxxxx … the explanation of the background work[1], the practical applications and the nature and purpose of the paper … xxxxx cccc (Tarantello, 1982a, b) xxxx ccc (Maschler and Peleg, 1976) xxxxxx.

2. This is a First-Order Title

2.1. This is a Second-Order Title

Consider an optimal control problem (OCP) with dynamics … xxxxxx cccccc.

Definition 1. A Borelian function $H\colon [0,T] \times \mathbb{R}^{2n} \to \mathbb{R}$ is called (A_∞, B_∞)-*subquadratic at infinity* if there exists a function $N(t,x)$ such that …

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This is a Third-Order Title. We shall first consider the question of nontriviality … xxxxxx cccccc xxxxxxxx ccc xxxxx ccccccc xxxxxx c xxxxx cccccc xxxxxxx.

Theorem 1 (Ghoussoub-Preiss). *Assume $H(t,x)$ is $(0,\varepsilon)$-subquadratic at infinity for all $\varepsilon > 0$, and T-periodic in t …*

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[1] See xxxxxxxx cccccccccc

This is a Fourth-Order Title. We assume that H is (A_∞, B_∞)-subquadratic at infinity ... ccccc xxxxxxxx ccccccc xxxx.

Lemma 1. *Assume that H is C^2 on $\mathbb{R}^{2n} \setminus \{0\}$ and that $H''(x)$ is non-degenerate for any $x \neq 0$. Then any local minimizer $\widetilde{x}$ of ψ has minimal period T.*

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Proposition 1. *Assume $H'(0) = 0$ and $H(0) = 0$. Set:*

$$\delta := \liminf_{x \to 0} 2N(x) \left\|x\right\|^{-2} \ . \tag{1}$$

If $\gamma < -\lambda < \delta$, the solution $\overline{u}$ is non-zero:

$$\overline{x}(t) \neq 0 \quad \forall t \ . \tag{2}$$

Proof (of proposition). Condition (1) means that ....

It is an exercise in convex analysis, into which we shall .... On the other hand, we check directly that .... □

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Corollary 1. *Assume H is C^2 and (a_∞, b_∞)-subquadratic at infinity. Let ...*

Xxxxxx ccccccc ... Proposition 1 tells us that ... xxxxxx ccccc xxxxxxxx ccccccc xxxx ccccc xxxxxxx ccccccc xxxxx xxxxxx.

2.2. This is a Second-Order Title again

The first results on subharmonics were ... xxxxx ccccccc xxxxx xxxxxx cccccc xxxx cc xxxxxxxxxxx ccccc xxxxxxxx.

Fig. 1. This is the caption of the figure displaying a white eagle and a white horse on a snow field.

Table 1. This is the example table.

Year	World population
8000 B.C.	5,000,000
50 A.D.	200,000,000
1650 A.D.	500,000,000
1945 A.D.	2,300,000,000
1980 A.D.	4,400,000,000

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Remark 1. The results in this section are a refined version of ....

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Example 1 (External forcing). Consider the system:

$$\dot{x} = JH'(x) + f(t) \tag{3}$$

where the Hamiltonian H is ... xxxxxx cccccccccc xxxxxxx ccc xxxxxxxxxxxxx.

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3. Conclusion

Xxxxxxxxxxx cccccccccc xxxxxxx ccc xxxxxxxxxxxxx (Clarke et al., 1980) xxxx cc-ccccccc. Xxxxxxxxxxx ccccccc ... by Subbotina (1986) ... cccc xxxx ccc xxxxxxxxx.

Acknowlegments. The authors express their gratitude to R. J. Brown for useful discussions on the subjects.

Appendix

1. First Appendix

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2. Second Appendix

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