

THE INTERNATIONAL MEETING **ISDG12-GTM2019**

July 03-05, 2019

St. Petersburg, Russia



ABSTRACTS

Edited by Leon A. Petrosyan and Nikolay A. Zenkevich

St. Petersburg State University
St. Petersburg
2019

ST. PETERSBURG STATE UNIVERSITY
THE INTERNATIONAL SOCIETY OF DYNAMIC GAMES

INTERNATIONAL MEETING ON GAME THEORY

*Joint meeting of “12th International ISDG Workshop”
(ISDG12) and “13th International Conference on Game
Theory and Management” (GTM2019)*

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INTERNATIONAL MEETING on GAME THEORY. Collected abstracts of papers presented on the international conference ISDG12-GTM2019 as joint meeting of “12th International ISDG Workshop” (ISDG12) and “13th International Conference on Game Theory and Management” (GTM2019) / Editors Leon A. Petrosyan and Nikolay A. Zenkevich. – SPb.: St. Petersburg State University, 2019. – 121 p.

The collection contains abstracts of papers accepted for the International Meeting on Game Theory (ISDG12-GTM2019, July 03-05, 2019, St. Petersburg State University, St. Petersburg, Russia). The presented abstracts belong to the field of dynamic game theory and its applications. The abstract volume may be recommended for researches and post-graduate students of management, economic and applied mathematics departments.

Computer design: Maria Bulgakova

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МЕЖДУНАРОДНАЯ КОНФЕРЕНЦИЯ ПО ТЕОРИИ ИГР. Сб. тезисов Международной конференции ISDG12-GTM2019 как объединенной конференции 12-го симпозиума Международного общества динамических игр и 13-ой Международной конференции по теории игр и менеджменту / Под ред. Л.А. Петросяна и Н.А. Зенкевича. – СПб.: Санкт-Петербургский государственный университет, 2019. – 121 с.

Сборник содержит тезисы докладов участников Международной конференции по теории игр (ISDG12-GTM2019, 03–05 июля 2019 года, Санкт-Петербургский государственный университет, Санкт-Петербург, Россия). Представленные тезисы относятся к теории динамических игр и её приложениям. Тезисы представляют интерес для научных работников, аспирантов и студентов старших курсов университетов, специализирующихся по менеджменту, экономике и прикладной математике.

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WELCOME ADDRESS

We are pleased to welcome you at the “International Meeting on Game Theory” (ISDG12-GTM2019) as joint meeting of “12th International ISDG Workshop” and “13th International Conference on Game Theory and Management” which is held in St. Petersburg State University and organized by the St. Petersburg State University (SPbSU) in collaboration with the International Society of Dynamic Games (ISDG).

One of the objectives of the International Society of Dynamic Games is to promote and foster the development and applications of the theory of dynamic games. A dynamic game is defined as a paradigm for a decision making situation, competitive or conflict process that involves several agents or decision makers in dynamic interaction. This objective is realized by the Society largely through the organization of regular international ISDG conferences and ISDG workshops, which alternate with periodicity once every two years.

The International Conference on Game Theory and Management (GTM) is designed to support further development of dialogue between fundamental game theory research and advanced studies in management. Such collaboration had already proved to be very fruitful, and has been manifested in the last two decades by Nobel Prizes in Economics awarded to John Nash, John Harsanyi, Reinhard Selten, Robert Aumann, Eric Maskin, Roger Myerson, Finn Kydland, Lloyd Shapley, Alvin Roth, Jean Tirole and few other leading scholars in game theory. In its applications to management topics game theory and theory of dynamic games contributed in very significant way to enhancement of our understanding of the most complex issues in competitive strategy, industrial organization, financial and operations management, to name a few areas.

Needless to say, that Game Theory, Theory of Dynamic Games and Management is very natural area to be developed in the multidisciplinary environment of St. Petersburg State University which is the oldest (est. 1724) Russian classical research University. This GTM conference was initiated in 2006 at SPbSU as part of the strategic partnership of it's the Faculty of Applied Mathematics & Control Processes and Graduate School of Management, both internationally recognized centers of research and teaching.

We would like to express our gratitude to the Conference's key speakers – distinguished scholars with path-breaking contributions to game theory, theory of dynamic games, economic theory, and management – for accepting our invitations. We would also like to thank all the participants who have generously provided their research papers for this event. We are pleased that this Conference has already become a tradition and wish all the success and solid worldwide recognition.

Leon A. Petrosyan, Co-chair of Program Committee

Nikolay A. Zenkevich, Co-chair of Program Committee

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A Multistage Hierarchical Non-Zero Sum Game of Competition on the Telecommunications Market

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Keywords: *hierarchical game, non-zero sum game, multistage game, subgame perfect equilibrium, competition.*

The paper investigates the process of competition on the market of telecommunication services between three firms: the leader, the challenger and the follower. In this work we construct a model of competition between three players in the form of a multistage hierarchical non-zero sum game and compare it with the model of competition between three players in the form of a multistage non-hierarchical non-zero sum game. As a solution of the game we find a subgame perfect equilibrium. We illustrate the results with an example for three companies working on the Saint-Petersburg telecommunications market.



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Game Theoretic Models in Marketing Networks

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Keywords: *social networks models, marketing, game theory.*

Models of influence in social networks are widely spread in the recent decades. The main result is that stable final opinions of all agents are determined only by the initial opinions of the members of strong subgroups in the influence digraph. This fact allows for an essential decrease of marketing cost when one or several competitive firms try to change the final opinions of the target audience. The respective game theoretic models are considered.



Journals in Game Theory

DYNAMIC GAMES AND APPLICATIONS

Editor-in-Chief
Georges Zaccour

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Several Model of Interaction of Consumers and Load Serving Entity in Retail Electricity Market

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Keywords: *load-controlled consumer, load serving entity, coordination of interaction, adverse selection model, demand side management.*

We consider the coordination of interaction between various types of consumers and a load serving entity to manage electricity consumption by using several models: the model of pricetaking, the Nash equilibrium pricing, and the adverse selection model based on the contract theory. We propose a method to form rate options for load curve optimization for different types of consumers and a load serving entity for different market configurations. The utility functions describe the real situation sufficiently well and allow the implementation of a system of incentives for load curve optimization (load shifting from a peak time of the day). The rates providing a separating equilibrium are determined. We compare the effectiveness of different retail market models for demand side management. One of the goals is the peak load reduction with respect to the average value during daytime. We use the pricing scheme that implies the change of electricity prices depending on the electricity consumption by all users during every hour, so that all users are financially motivated. We considered three possible models describing the retail market and compare their efficiency. Each model has its reasons to be used. The general equilibrium model realizes an effective scheme for all participants who do not have incentives to change their behavior (stable equilibrium) [1]. However, this model has high demands for the quality of information. The model with the individual rationality constraint is less demanding for consumers and every consumer receives the same profit. This model realizes the welfare maximum but does not provide a stable equilibrium. [2] The last model describes the situation of a possible arbitrage between users [3]. We consider a situation when some users switch to the rate of other users, thereby increasing their utility and reducing the retailer's profit. We propose a model that takes this effect into consideration, partly compensating for the loss of the LSE. The models have been tested on the data of several users from the student campus. Supported by grant N019-010-00183 from Russian Foundation for Basic Research.

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Coordination Games on Directed Graphs

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Keywords: *coordination games, strategic games on graphs, Nash equilibria, strong equilibria.*

In [1] we studied natural strategic games on undirected graphs. In these games the players are nodes in a graph, each with a finite set of strategies, with the payoff of each player defined as the number of his neighbours who selected the same strategy. These games capture the idea of coordination in the absence of globally common strategies. They always have (pure) Nash equilibria and the focus of this work was on identifying classes of graphs for which strong equilibria exist.

We pursued this work in a number of publications, more recently by considering directed weighted graphs. For such graphs the payoff is defined as the sum of the weights on the edges from players who chose the same strategy, augmented by a fixed non-negative bonus for picking a given strategy.

In these games Nash equilibria do not need to exist. However, for several natural classes of graphs, including cycles, open and closed chains of cycles, DAGs, and cliques, finite improvement or coalition-improvement paths of polynomial length always exist, and, as a consequence, a (pure) Nash equilibria or a strong equilibrium in them can be found in polynomial time.

For simple cycles these results are optimal in the sense that without the imposed conditions on the weights and bonuses a Nash equilibrium may not even exist. In the case of closed chains of simple cycles unweighted edges have to be used. In the restricted case of a DAG or a clique strong equilibria can be found along every coalition-improvement path.

The problem of determining the existence of a Nash equilibrium or of a strong equilibrium in these games is NP-complete already for unweighted graphs and no bonuses.

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Mean Field Type Differential Game

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Keywords: *mean field type differential games, feedback strategy, nonsmooth analysis, viability properties.*

We study the mean field type differential game with the dynamics of each agent given by

$$\begin{aligned} \frac{d}{dt}x(t) &= f(t, x(t), m(t), u(t), v(t)), \\ t \in [0, T], \quad m(t) &\in \mathcal{P}^2(), \quad u(t) \in U, \quad v(t) \in V. \end{aligned} \tag{1}$$

Here $m(t)$ is the distribution of all agents at time t ; $u(t)$ (respectively, $v(t)$) is the control of the first (respectively, second) player acting on the agent; U (respectively, V) is the control space for the first (respectively, second) player.

We assume that the players influence upon dynamics of each player independently. The purpose of the first (respectively, second) player is to minimize (respectively, maximize) the functional

$$g(m(T)).$$

The standard assumptions on the control spaces, dynamics and reward function are imposed. Notice that they include the Isaacs' condition.

The main results are as follows.

- We introduce the notions of u - and v -stability for this type of differential games. This conditions are close to the viability property in the space of probability measures on the phase space. It is proved that the upper value of the game is bounded from above by any u -stable function, whereas the lower value function is not less then every v -stable function.
- We prove the existence of the value function. This function is simultaneously u - and v -stable.
- We find the infinitesimal form of u - and v -stability conditions using an analogs of directional derivative. This provides the characterization of the value function of the mean field type differential game in the terms of non-smooth analysis.

Nash Equilibria in One-Memory Strategies

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Keywords: *1-memory strategies, reactive strategies, Nash equilibrium, zero-determinant strategies.*

We study the Nash equilibrium in infinitely repeated 2x2 games where limit-of-means payoffs are determined by reactive or by 1-memory strategies. Reactive strategies are stochastic 1-memory strategies such that a probability of players' actions depends only on the opponent's preceding move. We provide a characterization of all Nash equilibria in the class of reactive strategies and suggest a very intuitive geometric interpretation of them. We then derive a characterization for all symmetric stage games admitting Nash equilibria in the class of reactive strategies. We show that all (1-memory) strong mixed equilibria in a discounted game are generated by equalizer zero-determinant strategies. This allows us to compare our results with existing results for 1-memory strategies. Namely, we show that payoff relevant indeterminacy holds true and there is no folk theorem. Then we obtain a probability for an arbitrary symmetric game to have a Nash equilibrium in the class of reactive strategies. In contrast to existing results for 1-memory strategies, we demonstrate that the reverse dominance condition does not influence the existence of Nash equilibria.



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Conditionally Coordinating Contracts in Supply Chains

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Keywords: *coordination problem, supply chain, coordinating contract, game-theory approach, sales rebate contract, buyback contract.*

Coordination is an essential mechanism of supply chain management. One of the instruments implemented for the achievement of supply chain coordination is contracts. This study investigates the problem of supply chain coordination through two commonly-used types of contracts: sales-rebate and buy-back. The problem solution is based on the game-theory approach. According to this approach, a contract is coordinating if it satisfies two conditions: collective and individual rationality so it should be Pareto-optimal and a Nash equilibrium. It is proved in the paper that both considering contracts: sales-rebate and buy-back cannot coordinate the supply chain, since the property of individual rationality is not fulfilled for the supplier. Nonetheless, the authors argued that a conditional coordination providing the maximum of expected supply chain and retailer's profits and a higher supplier's expected profit in comparison with the wholesale price contract could be achieved under certain restrictions on the contract's parameters. For sales-rebate and buy-back contracts, the algorithm for constructing a conditional coordinating solution was developed, which includes finding the optimal retailer's order quantity; identifying the maximum of the supply chain expected profit function, obtaining a coordinating condition for the contracts parameters; and finding the restrictions on the parameters which provide the conditional coordination. The proposed algorithm was considered step in details for both types of contracts under assumption of uniformly distributed demand. In the paper examples of the conditionally coordinating contract design problem solving are also presented. These examples show that the choice of contracts parameters ensuring supply chain conditional coordination from a set of all possible coordinating parameters obtained in the optimization problem solving should be a result of the compromise finding through players' negotiation process.

Delegation as a Signal to Sustain Coordination: An Experimental Study

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Keywords: *delegation, Bayesian games, forward induction, experiment.*

This paper explores a potentially important role of delegation: as a signal of trust that is reciprocated by more cooperation. I consider a static principal-agent model with two tasks, one of which requires cooperation between the principal and the agent. If there is asymmetric information about the agent's type, the principal with a private belief that the agent is a good type can delegate the first task in order to signal the agent about his 'trust'. This equilibrium is supported by the forward induction argument. I conduct laboratory experiments to test these theoretical predictions and to examine the role of information in equilibrium selection. I find that delegation is used only sometimes to facilitate cooperation; however, when the subjects have information about past sessions, there is a statistically significant increase in the use of delegation. This evidence suggests that experience matters in equilibrium selection in Bayesian games.



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Gabasov's Approach for Nash Equilibrium Point in One Actuarial Problem

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Keywords: *local Nash equilibrium, Gabasov's adaptive method, Actuarial problem.*

In this paper we construct a non-cooperative game-theoretic model describing the interaction process of insurance companies in the CMTPL business line. As an optimality principle in the game we use a local Nash equilibrium [1], in order to define it the method of reduction to the linear programming problem and the adaptive method by R.Gabasov [2] are used. Corresponding algorithm is implemented in MATLAB, statistical data is used for a numerical simulation.

Let's consider a game-theoretic model describing the process of direct compensation of losses between insurance companies. Players are the insurance companies that make decisions on claiming losses during one settlement session, therefore we consider a static game. The process of exchanging the requirements evolves in the following way: if an insured event occurs with two participants, the victim can apply not only to the insurance company where he is a client, but also to the company of the other participant, in this case the insurance company which the victim turned directly pays the losses to the victim. Later on, the insurance company should demand compensation from the company of the culprit. The Belgian system [3] is used to exchange the requirements. During the settlement session (one week), insurance companies exchange requirements and based on the settlement session results, the average is calculated and all claims are refunded according to it. Since the requirements have different sums, the calculating process of the average sum of all session requirements can be adjusted, in order to obtain a large profit during the session. For simplicity, we consider one session and two insurance companies or static two player game.

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New Emergent Dynamics and Equilibria Types in Multi-modal Differential Games with Applications.

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Keywords: *differential games, sliding dynamics, piecewise-smooth systems, multi-modality, asymmetric players.*

This paper explores what new types of strategies emerge in game-theoretic setting with multiple regimes of dynamics of a state of the system. It is demonstrated, that resulting equilibrium trajectories are an emerging property of multi-modality of the game and new types of equilibria are studied. I first establish minimal requirements for a game to exhibit such a complex behavior.

In particular it is demonstrated that even the linear-quadratic type game with one state and two players may result in chaotic dynamics or limit cycles as well as pseudoequilibrium at the boundary.

These call to the formulation of an emergent secondary game around the switching manifold with new weaker requirements on the equilibrium.

Such a secondary game is formulated in terms of zero divergence of players' trajectories and it is shown to be individually and socially improving over the outcome emerging from the baseline game (i. e. non-deterministic chaos).

This theoretic setting is then applied to several relevant economic problems, such as resource management, advertising, R&D games and etc.



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SPRINGER



Learning under Partial Cooperation and Uncertainty

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Keywords: *uncertainty, learning, environment, dynamic game.*

We study a dynamic game of pollution control in a situation where the effects of greenhouse gases on global warming are not completely known. More precisely, we assume that there is uncertainty about the impact of accumulated pollution on the environmental damage cost. Countries are however able to increase their knowledge by using a Bayesian learning process, on the basis of their observation of the actual damages they incur. We study the consequences of uncertainty and learning in terms of pollution emissions and welfare.



Journals in Game Theory

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The Operator Approach to Dynamic Strong Stackelberg Equilibria

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Keywords: *game theory, Stackelberg equilibria, value iteration.*

The concept of Strong Stackelberg Equilibrium has received a renewed attention in the recent years, due notably to its application to security questions [1]. In Stackelberg games, it is assumed that one of the players, called the Leader, commits to a strategy. The other player, called the Follower, learns or observes this strategy, and reacts rationally to it. The game being with complete information, the Leader is able to predict the reaction of the follower, and is then able to optimize her strategy to maximize her own rewards. It may be that the follower's response is not unique, in which case this prediction of the leader is not possibly anymore. A strong Stackelberg Equilibrium (SSE) occurs when the Follower breaks such ties in favor of the Leader : she chooses the best option to for the Leader among the best ones for her. Initially defined in static games, this concept has been extended to infinite-horizon, discounted dynamic games. Calculating dynamic SSE in such games within the set of general strategies appears to be very difficult in general [2], with the feature that stationary strategies are suboptimal in general [3]. With the hope of reducing this complexity, as well memory storage for the strategies, many authors concentrate on stationary feedback policies. Even in this restricted class, the problem is still NP-Complete to compute in general [2] and several algorithms based on Mathematical Programming have been proposed in the literature for solving or approximating this problem [3].

In this paper, we study an alternative approach, inspired from the classical operator theory for Markov Decision Processes or Zero-Sum Nash Games. Our contribution is as follows : we first define a dynamic programming operator T corresponding to one-step strong Stackelberg games with a scrap value. Fixed-Point Equilibria (FPE) are defined as fixed points of T . The Value Iteration algorithm is defined as the iterations of T until convergence.

Part of this research was realized during the SticAmSud project 16-STIC-10 DyGaMe.

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A Differential Game of Pricing and Advertising for National and Store Brands

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Keywords: *differential games, marketing, pricing, advertising, private labels, feedback Stackelberg equilibrium.*

This paper deals with the issue of price and non-price marketing decisions for national and store brands. We use a differential game to investigate two scenarios: (1) a retailer that distributes only a national brand, and (2) a retailer that introduces a store brand. We compute strategies and profits under both scenarios and provide answers to the following research questions:

- 1) what should be the prices and the non-price strategies before and after the introduction of the store brand?
- 2) how do strategies and profits compare?
- 3) under which circumstances is the introduction of a private label?



Periodicals in Game Theory

GAME THEORY AND APPLICATIONS

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NOVA SCIENCE



International Trade in Monopolistic Competition framework: Social Optimality Case

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Keywords: *monopolistic competition international trade, social optimality, comparative statics.*

We study the homogeneous model of international trade under monopolistic competition of producers. The utility function assume additive separable. The transport costs are of “iceberg types”. Usually the equilibrium situations are considered. Unlike oligopoly, the game goes among infinitely many players and thereby strategic interactions are excluded in free-entry Nash equilibrium (each producer chooses her output, perceiving others’ strategies as given). It is known [1, 2] that in the situation of market equilibrium, under linear production costs, the social welfare as function of transport costs, decreases near free trade while (counter-intuitively!) increases near total autarky. Instead, we study the situation of social optimality. It can be interpreted as the problem of “social planner” who optimize a scalarization of multi-criteria to find Pareto-optimal solution. We show that the total welfare decreases. We restrict our study by the case of two countries. Moreover, we study two “limited” situations: near free trade and near total autarky. We show that near free trade, the welfare in small country decreases; as to the large country, we find examples when the welfare decreases and when the welfare (counter-intuitively!) increases. Besides, in the autarky case, we describe the situations of decreasing/increasing of welfare in each country. Supported in part by RFBR, projects 18-010-00728 and 19-010-00910.

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How Much We Trust What We See in Others, When We Are Different?

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Keywords: *two-population evolutionary game, heterogeneous preferences, imitative revision protocol.*

We analyze the interaction between individuals belonging to two different populations. They choose among the same alternatives (share the same strategy set), but they get different rewards depending on the population they belong to (different payoff matrices). In consequence, they face different dilemmas. We analyze a two-population evolutionary game, although we depart from the standard formulation as we allow agents in one population to interact with agents both within their own and also from the other population. Moreover, considering an imitative revision protocol, a revising agent can be paired with and imitate an agent belonging to his own population. Similarly, we also allow for the possibility of being paired with someone from the other population. When this latter is playing an alternative strategy, he obtains a payoff different from the payoff estimated by the revising agent. Now, the revising agent is uncertain on the payoff he will obtain by switching to this alternative strategy. He must assign a probability to the payoff he observes in his pair versus the payoff he estimates from his payoff matrix. This probability determines to what extent the revising agent behaves erroneously. Yet, under this behavior we prove the existence of a unique mixed-strategy asymptotically stable fixed point of the evolutionary dynamics. We also analyze how the asymptotically stable equilibrium is affected by the probability that agents assign to what they see when paired to someone from a different population.



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Games with Information. Introducing Witsenhausen Intrinsic Model.

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Keywords: *game theory, information, extensive form.*

In a context of competition, information — who knows what and before whom — plays a crucial role. Here, we concentrate on three models (two in game theory, one in control theory) where the concept of information is present: Kuhn's extensive tree model (K-model), Alos-Ferrer and Ritzberger infinite tree model (AFR-model) and Witsenhausen model (W-model).

The model proposed by Witsenhausen has the following main ingredients: a set of agents taking decisions in a decision space, Nature taking decisions in a sample space, configuration space which is the product of the decision spaces by the sample space, information fields that are sigma-fields on the configuration space and strategies that are measurable mappings from configurations to actions w.r.t. information fields. W-model deals with information in all generality. Thus, it allows to look at a problem without a priori knowing the order in which decisions were made by agents. In the subclass of causal systems, there is at least one ordering in which agents take their decisions consistently with the given information.

The extensive form is the most richly structured way to describe game situations. In his model, Kuhn uses the language of graph theory to define main ingredients of the game: players, game tree, information sets and strategies. The infinite tree AFR-model generalizes K-model to any possible tree: infinite (repeated games), transfinite (long cheap talk) and even continuous (stochastic games). The authors develop the refined partition approach that proves to be useful for giving a simple equivalent condition for a player to have perfect recall. To tame the zoo of infinite trees, authors use the language of set theory constructing not trees, but posets, thus elaborating the most general framework to describe tree structures in games existing up to now.

We study whether AFR and W-models have the same potential to model games. First, we embed the subclass of causal W-models into the AFR-formalism by building an AFR-tree and translating definitions of information and strategies from W-formalism to AFR-formalism. Second, we move in the opposite direction. All along, we provide illustrations of the power and simplicity of the W-model to handle information in a strategic context.

A Note on the Role of Value Heterogeneity in Two-way Flow Strict Nash Networks with Small Decay

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Keywords: *agent heterogeneity, information network, network formation, strict Nash network.*

Nonrival information refers to information that agents do not mind sharing with each other. Each piece of information that an agent possesses, though, is likely to vary in terms of value. Such variation in terms of information value can depend on either the identity of agent who possesses it (i.e., some agents may own pieces of important information compared to others, or we call this partner heterogeneity) or the identity of agent who chooses to retrieve it (i.e., the same information may have less value to some agents compared to others, or we call this partner heterogeneity) or both (we call this two-way heterogeneity). In this note, we study how this heterogeneity in the value of nonrival information impacts the shapes of Strict Nash networks (SNNs, henceforth). We do so by extending the work of Jaegher and Kamphorst (2015), which is a two-way flow model of network with small information decay. By allowing for the presence of value heterogeneity in this model, we study the changes in terms of shapes and other important properties of SNNs compared to original result of Jaegher and Kamphorst (2015) (Proposition 1 and 2) in which value heterogeneity is absent. We briefly review the literature and point out our contribution here. The literature that studies the game-theoretic model of the formation of two-way flow nonrival network originates from the seminal work of Bala and Goyal (2000). Two unique features of this model are that an agent can choose to form a link with another agent without mutual consent. Within this literature, extensive the studies on the role of agent heterogeneity in link formation cost have been done. Little is known, however, about the role of agent heterogeneity in information value. To the knowledge of the author, the only fact we know about the role of agent heterogeneity in information value is that it does not change the shape of SNN although disconnected components can emerge. This result is established as Proposition 3.1 in the work of Galeotti et al. (2006), which assumes the absence of information decay. This present paper, therefore, contribute to this body of literature by studying the roles of agent heterogeneity in information value in the presence of small information decay rather than being completely absent in the previous literature, where the term ‘small’ here refers to the fact that no superfluous link is worth establishing by an agent.

Recent Studies of Agent Incentives in Internet Resource Allocation and Pricing

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Keywords: *market equilibrium, auction, Nash equilibrium, truthfulness, incentive ratio, algorithmic game theory.*

Market makers choose and design market rules to serve certain objectives, such as to maximize revenue from the sales in the case of a single seller and multiple buyers. Given such rules, market participants play against each other to maximize their utility function values on goods acquired, possibly by hiding or misrepresenting their information needed in the implementation of market rules. Today's Internet economy has changed the information collection process and may make some of the assumptions of market rule implementation obsolete. Here we make a fresh review of works on this challenge on the Internet where new economic systems operate.



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Games with a Fuzzy Payment Matrix

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Keywords: *a zero-sum game, fuzzy number, fuzzy payment matrix, pure strategy.*

A large number of tasks related to the study of operations in a formalized form have a matrix representation. The most common elements of the payment matrix are represented as point numbers in different numerical scales. Classical game theory is based on the assumption that players have complete information about the payment matrix. However, this assumption is not realized in practice. The exact definition of the elements of the payment matrix is difficult because of the inaccuracy of the original information or the inaccuracy of the model of the system under study. In addition, quite often the elements of the payment matrix are set by expert. Expert assessments are fundamentally characterized by uncertainty. In this regard, the situations when the elements of the payment matrix are fuzzy numbers or linguistic estimates [1-6] are considered more and more often [1-6]. When building a fuzzy payment matrix, the elements of which will be fuzzy numbers need to decide on the type of membership function of a fuzzy number, and how to determine the width of its carrier. Taking into account the known requirements for membership functions of fuzzy numbers can be selected linear-piecewise, Π -shaped or bell-shaped functions that can be used to simulate different levels of uncertainty in the evaluation of the elements of the payment matrix. In addition, it is necessary to decide whether to use one type of membership functions for all elements of the payment matrix or different. Apparently, the main criterion for solving this problem should be considered the complexity of the required computational procedures and the conditions for their correct application[6]. When selecting the width of the fuzzy number media, there are also options, but the simplest is to determine the width of the media as a percentage of the modal value. In the future, for simplicity, we will focus on the triangular membership function of fuzzy numbers for all elements of the payment matrix.

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Communication in Mixed Oligopolies

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Keywords: *mixed oligopoly, information sharing, stochastic demand.*

In this study, we examine incentives for public and private firms to cooperate in information sharing about stochastic market demand. An advantage of cooperation in information sharing is that all firms predict environment and market changes better, which may increase production and allocative efficiency. On the other side, there are competition disadvantages of sharing information with competitors. It will be easier for companies to monitor each other when they have more detailed information about their competitors. Specifically, we model an environment where a private and a public firm competes in quantities (à la Cournot) and each receives noisy signals on stochastic demand. Then, we derive Perfect Bayesian Equilibrium of the game and analyze at which degree these firms cooperate and share their private signals with each other while continuing to compete in quantities. We establish that firms completely share their private signals with each other if the goods are complements. Hence, there is full cooperation in this case. If the goods are substitutes, while private firm does not share any information, public firm continues to share its private information completely with the private firm.

This result indicates that for the social welfare maximizing public firm, benefit of reducing demand uncertainty in the market strictly dominates benefit of competitive advantage that the private information provides. Although firms are allowed to share their private information partially, partial information sharing never arises as an equilibrium.



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Management of a Charity: Optimal and Equilibrium Policies

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Keywords: *charities, dynamic optimization, dynamic games, management style, advertising, reputation.*

Nonprofit organizations play an important role in providing goods and services in all countries. In the first part of the paper, we consider one charity and determine its optimal policies. In the second part, we extend the model to two charities competing for donations and determine a feedback-Nash equilibrium.

The starting point of our analysis is that the amount of donations received by a charity are function of its reputation, which is an asset that can be built up over time, not overnight. To account for this important aspect, we propose a dynamic model where the charity can allocate its revenues to three main activities, namely, program expenses (charitable projects), information (promotion of its causes, website, etc.) and administration (worker/manager salaries and other administrative costs). We assume that the donors are sensitive to the way in which the charity is managed. If the administrative expenses are above a socially accepted norm, then the charity's reputation suffers. The opposite occurs when the charity is efficient and keeps its administrative costs below the norm. We prove that depending on the parameter values, there exist different optimal policies involving either positive or nil advertising and administrative expenses. We discuss some policy implications for each case and assess the impact of the norm on the results.

Finally, we compare the cases where the two charities cooperate and compete in attracting donations.



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Functional Analysis of the Business Games Objects

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Keywords: *business game, object characteristic, functional analysis, hypothesis, problem.*

The object is characterized by three parameters: change in the bid, change in the resource allocation, change in the profits. Each object is depicted as a point in three-dimensional space and forms a cloud of values.

The following hypotheses are put forward:

- 1) the functional dependence describing the upper boundary of the cloud characterizes the behavior model of the risk player;
- 2) the functional dependence describing the lower boundary of the cloud characterizes the behavior model of a cautious player;
- 3) the functional dependence describing the cross section of a cloud characterizes the behavior model of an average player.

Problems identified:

- 1) a large amount of data is needed;
- 2) the cloud upper boundary characterizes the behavior of not one person, but the general situation in the risky players group;
- 3) the cloud lower boundary characterizes the behavior of not one person, but the general situation in the cautious players group;

As a research method, functional analysis was used. We used results of games with mechanisms: Alternating direction method of multipliers (ADMM), Groves–Ledyard (GL),

Groves–Ledyard reduced (GLR), Uniform mechanism (UNI), Yang and Hajek (YH).

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Modeling of Influence Among the Participants of the Educational Team

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Keywords: *degree of influence.*

In the report we will present the mathematical model of ranking of participants in the educational collective. This model is interesting as from a theoretical point of view as from a practical point of view. It can be used to determine the degree of influence of principals on any educational group at different levels of education. Another practical aspect is the definition of professional orientation of students. We use the De Groot model in the modeling process.

The education group consists of students and teachers (principals). Principals are the centers of the educational team. In our model, the principals do not have influence on each other, but they have influence on students only. We represent principals as an association of teachers of various disciplines: natural sciences and humanities.

Principals have influence on students and vice versa. In our model the center of research is this feedback from students. We have defined this effect as the degree of the principal's influence.

The influence of the participants is described by a stochastic square matrix $\sum_{j=1}^n a_{ij} = 1$ and $A \in [0, 1]^{n \times n}$. In this matrix a_{ij} is the degree of influence of participant j to i ; n - is number of members in the team.

In our report we focus on the weight of the team members, which is expressed by the vector of limiting influence $A^t, t \rightarrow \infty$. The issue of opinion dynamics will not be considered. Students have different levels of training. All students can be ranked from the best to the worst. According to their rating students can be divided into m subgroups. In each subgroup students have the same rating.

Important parameters are the influence of the principal on members of the group $\varepsilon \in [0, 1]$, and the influence of each member on the principals $\delta \in [0, 1]$.

In the report we will present the following scenarios:

1. The model with one principal and several subgroups of students.
2. The model with two principals and several subgroups of students.

The limiting influence vectors of the participant will be shown for both cases.

For persuasiveness and clarity we give a numerical example. We consider two real groups of Petrozavodsk state University, Institute of mathematics with different rating levels. There are two groups of teachers (natural sciences and humanities). They are principals. This numerical example demonstrates the process of finding of the vector limiting influence. We present significant conclusions about the limiting degree of principals influence in groups with different levels of training.

An Analysis of a Reflexive Control of the Tullock Contest

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Keywords: *social networks, game theory, uncertainty, epistemic, knowledge, reflexive control.*

We focus on the analysis of a reflexive control in the Tullock contest for two players [1]. The analysis is based on papers [2, 3] and have been already applied to some games [4]. The Tullock contest could be described by a set of agent $N = \{1, 2\}$ and with real not negative strategies and utility functions $f_i = x_i^\alpha / \sum_j x_j^\alpha - x_i$. We consider only games with $0 < \alpha < 1$. There is a well-known Nash equilibrium for this game [1]: $x_{1*} = x_{2*} = \alpha/4$, see [5].

In this paper, we consider α as an uncertain parameter for agents and they have to make some suggestion about it. Their suggestion could be different. So player 1 could believe that all utility functions are $f_i = x_i^{\alpha_i} / \sum_j x_j^{\alpha_j} - x_i$. It coincide with the Nash equilibrium with certain value of parameter α , if there is $\alpha = \alpha_i$ for any i common knowledge that $\alpha = \alpha_i$. If agents have no communication among them then they should compose and solve a system of equations. Strategy of each player is equals her best response and for all other agents agent i make a best response that is based on his own believe. Thus from the i -th player's point of view he should compose and solve system for following best responses for each j . We found that for this case partial derivatives are $df_1/d\alpha = (\alpha_1^{\alpha-1} \alpha^\alpha (\alpha (\alpha_2^\alpha - \alpha_1^\alpha) \log(\alpha_1/\alpha_2) + \alpha_1^\alpha + \alpha_2^\alpha)) / (\alpha_1^\alpha + \alpha_2^\alpha)^3$. It means that increasing α without changing epistemic representations of α in the agents' minds will lead to the increase of utility of that agent who consider α smaller than her opponent does. There could be a communication between agents but then trust each one only partially and they can communicate according de Groot model [5]. There is not difference if an existence of such communication is a common knowledge among all agents or it is not. Let their influences are x_j then one should compose and solve system of best responses for each i . If there is a communication with no trust at all, then all agents become stubborn and other opinion does not change their opinions though they have to be taking into account. There is not difference if an existence of such communication is a common knowledge among all agents or it is not.

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On the Existence of Location Nash Equilibria Under Delivered Prices and Price Sensitive Demand: An Empirical Study

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Keywords: *discrete competitive location, Nash equilibrium, delivered pricing.*

Many competitive location models attempt to find facility locations for competing firms with the aim of profit maximization. Profit depends on both the location of facilities and the prices offered by the firms in each customer area. If the firms enter simultaneously in the market, maximization of their profits can be seen as a two-stage game. In the first stage, the firms choose their facility locations. In the second stage, the firms compete on price. The two stage game can be reduced to a location game if there exists a unique price equilibrium in the second stage which depends on the facility locations.

With delivered pricing, customers buy from the firm that offers the lowest price and there exists a price equilibrium for any set of facility locations. At equilibrium, each competing firm monopolizes a group of markets, the ones to which the firm deliver the product at a minimum delivered cost. Then the location-price game is reduced to a single stage location game. Existence of a Nash equilibrium for this location game has been proved when demand is fixed in each market and it does not depend on the price. In such a case, a location Nash equilibrium is any set of facility locations which is a minimizer of the social cost.

When demand is price sensitive, to our knowledge, the existence of a location Nash equilibrium has not been proved in the location literature. The aim of this paper is to apply a mathematical programming model to find the best facility locations for one firm, assuming that the facility location of its competitors have been fixed. Then, the best response procedure is used to find a location Nash equilibrium, if there exists. An empirical study is presented where both markets and location candidates are Spanish municipalities. A variety of instances of the location game are explored to determine if the best response procedure converge, or not, to a location Nash equilibrium.

Welfare effects of Spoofing Manipulation Strategy in Financial Double Auction

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Keywords: *financial markets, market manipulation, order book modeling, high-frequency trading, double auction.*

Electronic markets with automation of trading and high speed trading have transformed the financial market landscape. Automated traders have ability to gather the market information from different sources including the order book, that represents the list of buy and sell orders for a specific security organized by price level. The development of electronic markets may contribute to market efficiency, whereas may also introduce new possibilities of disruptive practices, when trades make tremendous profits by artificially affecting market beliefs. The presence of market manipulation strategies represents an element of price distortion and creation of artificial market conditions. It represents major threat to trust and market integrity in capital markets through mispricing and market imperfections. It harms investors' confidence, resulting in less participation of investors, and adversely affects efficiency, liquidity, integrity and development of the stock market [1-3]. These negative practices have increased over the period of time and recently market fraud is the most serious issue faced by global markets, which constitutes 41% of overall frauds. Therefore, market manipulation is an important issue to detect and prevent across all of the financial markets including the US and the European and the emerging markets [3-6]. In the research spoofing manipulation is analyzed as one of the manipulative algorithmic trading entity in order to outperform other market participants by working in the order book. On the commodity and financial markets the order book represent trading mechanism, when both buyer and seller are submitting bids. Such mechanism is called a double auction, where both sides are competing, and is analyzed from the perspective of game theory. The investigation of the effect of spoofing manipulation strategy on market performance is done by modeling the order book as financial double auction.

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Optimal Advertising and Entry Timing in Markets with Peer Influence between Elites and Followers

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Keywords: *new products, advertising, entry timing, marketing strategy, optimal control, peer influence.*

We study the optimal advertising and entry timing decisions for a new product being sold in a segmented market where peer effects exist between an elite and a follower segment, elites may disadopt, and entry is costly. Followers are positively influenced by elites, whereas elites may be either unaffected or repulsed by followers. We develop a continuous time, optimal control model where the decision variables are the amount of advertising in each segment and the time of entry into the follower segment. Our analysis generates two sets of two-point boundary value problems where one set has an unknown boundary value condition that satisfies an algebraic equation. A fast solution methodology is proposed. Two main insights emerge from the analysis. First, in markets where repulsion slows adoption but does not induce disadoption, delaying entry contributes little to the bottom line if the firm already manages dynamic advertising optimally. In contrast, in markets with repulsion-induced disadoption, managing both advertising and entry timing leads to markedly higher profits than managing only one of these levers. Second, with repulsion-induced disadoption, the optimal advertising strategy in the elite segment is U-shaped, i.e., decreasing at first but then increasing again to thwart the repulsion influence of followers.



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Investment in Cleaner Technologies in a Transboundary Pollution Dynamic Game. A Numerical Investigation

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Keywords: *differential games, transboundary pollution, investment in cleaner technologies, numerical methods.*

Within a non-cooperative transboundary pollution dynamic game, we study the strategic impact of a region's investment in the adoption of a cleaner technology, as embodied by a reduction in the emission per output ratio, on the equilibrium outcomes and regions' welfare. The ratio of emissions to output is endogenous and is a decreasing function of the level of the stock of clean technology. Each region can invest in a clean technology in addition to its control of emissions. Clean technology is assumed to be public knowledge so that both regions benefit from the investment in clean technology of an individual region. Pollution damage is modelled as a strictly convex function in the pollution stock. We analyze the feedback equilibrium of the non-cooperative game between two regions played over an infinite horizon. The formulation of the transboundary pollution dynamic game does not fit any special structure of analytically treatable games such as linear-state or linear-quadratic differential games. We develop numerical methods to characterize the feedback equilibrium of the non-cooperative game between two regions. The equilibrium trajectories of the stock of pollution and stock of clean technology as well the regions' welfare are compared under different scenarios.



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DYNAMIC GAMES AND APPLICATIONS

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Institutions and Political Economy in a Capital Accumulation Growth Model

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Keywords: *differential games, dynamic programming, economic growth, political economy.*

A Markov Perfect Nash Equilibrium is obtained for a capital accumulation differential game that yields the market equilibria under different sequences of tax policies given by an institutional framework. The model aims to explain the effect of different institutional frameworks in economic growth under neoclassical assumptions of the market. The model consists on a set of workers that supply their work inelastically, a set of entrepreneurs with access to the technology of production and a set of elites, who may or may not engage in entrepreneurial activities. The participants on the economy play a game in which the entrepreneurs decide the level of production of the economy considering taxation levels and transferences made for each group. We find that the economic equilibria under inclusive political institutions provide an increase in total welfare in the long run. This is consistent with other findings in political economy and selectorate theory and can be used for the design of institutional rules that provide policies that guarantee long run sustainable economic growth.



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Epidemic Propagation in Growing Social Networks

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Keywords: *graph, propagation, social network, deactivation of nodes.*

There are many interesting research results in epidemic propagation, see SIS, SIR models [1], including analytical solutions for the complete graphs. New approaches involve combining two mainstream ideas [2-4] – epidemic propagation models [5] for static networks [6] and models of network growth [7]. We numerically compare the statistical properties of epidemic propagation on different types of static and dynamic networks, including the networks that are growing according to the model with deactivation of nodes [8]. There are no viruses but one can use the same models for marketing purposes as propagation of ideas or preferences [9]. The input data for the algorithm with deactivation of nodes: $Z > 0$ – the number of new active nodes on each step, $P \in (0, 1]$ – the probability of a new link, for each active node on each step, $G \in (0, 1)$ – the share of the active nodes which are deactivated on each step. On every step the following procedure is repeated until the desired number of linked nodes is reached: a) Z new active nodes are added; b) each active node with the probability P generates an undirected link with randomly selected another active node; c) a share G of the active nodes are deactivated which are selected with the probability in inverse proportion to their degree. The links and nodes are not deleted, only the activity status is changed which influences on the new links generation. For technical reasons when calculating degrees we assume that each node has a link with itself, so a degree never equals zero. Output data: an undirected network with statistical properties similar to online social networks.

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International Environmental Agreements: When is it Optimal to Join?

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Keywords: *IEA, differential games, optimal switching.*

The objective of this paper is to analyze the circumstances under which a country has interest to join -immediately or at some future point in time- an already signed International Environmental Agreement. For this reason, we consider a binding International Environmental Agreement (IEA), initially agreed by N countries (henceforth signatory countries), and a non-signatory country (henceforth defector), who contemplates when (and whether) it is optimal to join the agreement. With the IEA, signatory countries commit to choose their emission levels cooperatively. Signatories punish defectors. The decision problem can be cast as follows: Suppose that choice to adhere to the IEA is irreversible: once entered, signatory countries are unable to quit the IEA. Under which circumstances does the defector join immediately, delay adhesion, or remain defector forever? If the defector delays adhesion, what are the factor in uencing the delay? What is the impact of the optimal decision on the signatory countries?



Periodicals in Game Theory

CONTRIBUTIONS TO GAME THEORY AND MANAGEMENT

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Artificial Intelligence in a Supply Chain with Dynamic Inventory

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Keywords: *artificial intelligence, supply chain management, dynamic games, vendor management inventory.*

This paper investigates the impact that Artificial Intelligence (AI) has on a Supply Chain (SC) performance. The research starts by analyzing a traditional SC with one manufacturer and one retailer. The manufacturer decides the production rate to fulfill the retailer's inventory stock through a Vendor Management Inventory (VMI) system. The retailer sets the price for the final product, which has a negative impact on sales. The VMI system is governed by a state equation whose stock increases in the production rate and decreases according to the demand. The VMI system is influenced by a random component characterizing the forecast errors. In a benchmark scenario, we identify the impact of this error term on the firms' profits and strategies by using a dynamic game with feedback strategies. The outcomes of the benchmark scenario will be compared to another scenario in which the manufacturer invests in an AI system. One of the main properties of AI is that it allows the SC to remove the forecast errors through a Watson AI system (e.g., Watson Supply Chain Insights from IBM). The benefits of removing the forecasting error term will be compared to the AI investments efforts and the newsfeed routine costs. The findings aim at highlighting the conditions under which SC members should engage in the implementation of an AI system and its impact on the SC relationships and structure.



Journals in Game Theory

INTERNATIONAL JOURNAL OF GAME THEORY

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SPRINGER



Trust, Altruism, Reciprocity: A Possible Solution to the Problem of Trust in Online Marketplace

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Keywords: *differential mechanism design, sharing economy, reciprocal rating.*

The problem of finding methods to establish trust and reputation in an online marketplace has attracted many researchers for the last two decades, as the transactions in online marketplaces are anonymous, geographically sparse, and sequential in nature. Various forms of feedback system has been designed to ensure the accuracy of a review, portray the proper reputation, and maintain the balance of information between various stakeholders in the marketplace in order to maintain a certain amount of trust within it. Of all these forms of feedback, Reciprocal Review system, where buyer and seller both rate each other and the aggregate rating is subsequently used as a proxy for reputation (or 'willingness to buy') in the market, was thought to be one of the finest mechanisms to ensure trust and accuracy of the system for its unique design of cooperation. Companies like eBay, Uber, Lyft, Airbnb, had adopted the Reciprocal Review system to establish trust among the buyer and the seller, but it was seen that Reciprocal Review system was not able to provide an accurate picture of the marketplace in terms of the service quality. Reciprocity was found to be extremely high in this system of review/rating, and thus the average ratings were found to be significantly higher than that were found in other similar marketplaces.

In this paper, for the first time in the study of shaping digital reputation, I have tried to design a system where I have shown that establishing trust and ensuring reputation can be possible within the domain of Reciprocal Review. I have developed a game theoretic model of the review/rating mechanism and shown that the equilibrium of the proposed system weakly ensures accurate review from both the parties in the marketplace, thus ensuring the level of trust in the market.

A Substitute for the Classical Neumann-Morgenstern Characteristic Function in Cooperative Differential Games

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Keywords: *differential games, characteristic function, Neumann-Morgenstern approach.*

It is common to describe the cooperative game in the form of characteristic function which shows the “power” of the coalition S . For dynamic games the method of construction of the characteristic function becomes crucial primarily due to technical complexity. The new method of characteristic function construction is introduced which is much more easily technically. The properties of a new characteristic function are investigated and it is proved that in many applications it can be used instead of classical Neumann-Morgenstern characteristic function.



Journals in Game Theory

GAMES AND ECONOMIC BEHAVIOR

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Overview of Some Methods Used in Solving Differential Games

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Keywords: *differential games, linear-quadratic games, LQR methods, selection of Nash equilibria, characteristic function, n-person game, hybrid games, Pontryagin maximum principle.*

Differential games describe a wide class of conflict problems in which the controlled by several participants process evolves in continuous time and its dynamics is described by system of ordinary differential equations. The main drawback of such class of problems is that the results very seldom may be obtained in the analytical form (and that is why is not applicable). In the presentation we will present some methods which we collected during solving the problem of common pollution control, resource extraction, goodwill accumulation etc. This methods could be applied for solving both for differential games with special structure (linear-quadratic, logarithmic, hybrid) and differential games in general form. 1) We suggest matrix transformation of utility function for linear-quadratic case to the only quadratic form which simplify the next steps of optimal control calculations. 2) For the case of non-uniqueness of the solution of non-cooperative game (Nash equilibrium) we demonstrate 2 methods of selection based on classical LQR analysis and an economic criterion described by Bass, Krishnamoorthy, Prasad, and Sethi in 2005. 3) For cooperative form of the game we present several methods of characteristic function values calculations and described approach which will help to obtain results for n players game (for any n). 4) Finally we suggest an approach of solving differential games with hybrid structure in which the utility function may change in the fixed time instants. The results of Pontryagin maximum principle are used.

Evolutionary Model of Virus Competitions in Networks

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Keywords: *evolutionary game, network security, epidemic process, optimal control, scale-free networks.*

Complex networks can be used to describe a large amount of biological, social and communication systems. As well as biological systems, computer networks also can be attacked by viruses (malicious software) and the process of diffusion of malware resembles epidemic process in biological populations. However the structure of network provides some restrictions in this diffusion process. In this work, we examine epidemic process of virus spreading on scale-free networks, where nodes represent individuals and edges mean interactions between them. We use Barabási–Albert algorithm for generating random scale-free network. It involves two important general concepts. First, number of nodes in the network are increasing over time. Second, the more connections node has, the more likely it is to receive new links. According to classical Susceptible Infected Recovered (SIR) model we set that each node can be in different states such as Susceptible node has not been infected yet, Infected node has been attacked by virus and Recovered node has been recovered. We also suppose that to describe real life process we need to suppose that several malwares circulate over the network. Hence the main difference from the classical model is the presence of the several competing viruses with different strength in the network then group of infected nodes splits into several subgroups.

We assume that nodes with higher degree have higher probability to form new links. Infection rate between nodes depends on its neighbors, which mean that, the strength of virus is defined by a number of captured nodes. In this study we design series of simulations where we run epidemic process with different variations of parameters over constructed networks and estimate the trends in diffusion of viruses and corresponding treatment strategies. We formulate an optimal control problem that aimed to eliminate the malwares from the network.

Two-Step Pricing Game

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Keywords: *price games, Nash equilibrium, pure strategy.*

Let $G = \langle V, E \rangle$ be an undirected graph, V — set of vertices, E — set of edges. On the graph G consider the pricing game. Let N be a set of players. The player's strategy is to select a subset of V vertices and set prices at selected vertices.

Introduce the notation: c_i^v is the cost of the player i , $i \in N$ for the implementation of items in the vertex v , $v \in V$, $c_i^v > 0$; p_i^v is the price at which player i is selling the item at the vertex v , $p_i^v \geq c_i^v$; a_i^v is the quality factor of the product of the player i at the vertex v , $a_i^v > 0$; $e^{-a_i^v p_i^v}$ is proportion of consumers at vertex v that are ready to buy a product of the player i , assuming that player i the only one on the market.

Let players from the set K , $K \subseteq N$, $K \neq \emptyset$ choose the vertex V . Then the product

$$\prod_{l \in L} e^{-a_l^v p_l^v} \cdot \prod_{j \in K \setminus L} (1 - e^{-a_j^v p_j^v})$$

is the proportion of consumers at vertex v which prefer products from a variety of L , $L \subseteq K$ and do not prefer products from a set $K \setminus L$. If $L = \emptyset$, we define the product $\prod_{l \in L} e^{-a_l^v p_l^v}$ identically equal to one. Similarly, if $K \setminus L = \emptyset$, then $\prod_{j \in K \setminus L} (1 - e^{-a_j^v p_j^v}) \equiv 1$.

Denote $h_i^v(p^v(K))$ the net profit of the player I at the top of v , then

$$h_i^v(p^v(K)) = (p_i^v - c_i^v) \cdot d_i^v(p^v(K)). \quad (2)$$

Each player $I \in N$ chooses a subset of vertices $s_i \subseteq V$, $s_i \neq \emptyset$ in which to trade. Then the functions of the winnings, players will take on the appearance

$$H_i(s, p) = \sum_{v \in s_i} h_i^v(p^v(K)), \quad (4)$$

Let's denote $k^v(s)$ the number of players who chose the vertex V . Then the winning function of the players in the first step at the optimal values of prices will take the form

$$H_i(s, \bar{p}) = \sum_{v \in s_i} \frac{1 - (1 - A^v)^{k^v(s)}}{a_i^v \cdot k^v(s)}$$

Theorem. *On the first step, there is a Nash equilibrium in pure strategies.*

A Model of Competition between E-commerce Platforms

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Keywords: *E-commerce platform, network externality, switching cost, first-mover advantage.*

In this paper we build a model of competition between e-commerce platforms. In the first stage of the game, there is an incumbent monopoly service provider, and it faces competition from an open-source platform in the second stage. Through model solving and numerical simulation, we find the following results: when the network externality is large, the first mover advantage formed from switching cost and product upgrade will lose effect. The first-mover platform should set a low price to attract more users in the first stage and choose an open policy in the second stage in order to compete with the open-source platform.



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Cooperative Working Capital Cost Game with Coalitional Structure for Distributive Supply Network

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Keywords: *working capital management, supply chain finance, cooperative game with coalitional structure.*

Working capital management (WCM) is increasingly recognized as important means of liquidity and profitability improvement (Talonpoika et al., 2016), specifically in terms of globalization and growing competition between supply chains. At the same time, rising financial risk in supply chains (SCs) stimulated management to recognize that the financial side of supply chain management (SCM) is a promising area for improvements. Nevertheless, companies still focus on their individual SC issues and take their own interests into account rather than understanding the whole SC and cooperating with their partners (Wuttke et al., 2016). Authors address this gap by developing cooperative game of working capital management aimed at minimizing total financial costs associated with each SC stage. The model is verified on the grounds of the combination of mathematical modeling and case study of Russian collaborative SC. The suggested model provides financial illustration for the motivation of SC partners to cooperate in order to simultaneously achieve target levels of working capital investments and improve individual financial performance through collaborative actions. As a result, the whole SC competitiveness and value is significantly strengthening. We develop a model that analyses working capital management process for 3-stage supply network. The focal network is a distributive supply network consists of N suppliers, one distributor and M retailers connected through material, information and financial flows. The members of the network can form various coalitions with the distributor. Each member's working capital position is constrained by liquidity and profitability requirements. As such, they face the need to control and manage financial costs associated with each stage. We construct characteristic function of each coalition as a minimum value of the sum of financial costs associated with working capital allocation. For this cooperative game with coalitional structure we investigate such optimal imputations as Shapley, Owen and Aumann-Dreze values and provide their comparative analysis. Theoretical results are illustrated with the numeric example of a real-life supply network.

This work is supported by the Russian Foundation for Basic Research, project N 17-07-00371.

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Role of Information in Decision Making in «The Price is Right»

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Keywords: *optimal stopping, n-person game, threshold strategy, showcase showdown.*

In this work, a game-theoretic approach is used to determine optimal behavior of the participants of the game show “The Price is Right”. We consider the following non-cooperative n -person optimal stopping game. A player receives scores by observing sums of independent and identically distributed random variables. At each step, each player can decide whether to stop on the current observation or to continue the game and receive the next value of independent random variable, which is added to the scores obtained previously. The winner is the player whose total scores is the closest to, but not exceeding the given upper limit. In case all the players exceed the given limit, then the winner is the player with the lowest number of scores. Each player aims to maximize his own winning probability. According to the information type, the two versions of the game are considered: a version with no information, where player makes the decision without information about behavior of other players, and, a full-information version, where each subsequent player is aware of actions and outcomes of previous players. Moreover, the players can be given an opportunity to receive their scores not only from two steps, as in TV show, but also from multiple steps, more than two. The game without information was considered in [1, 2]. The problem with full information and two steps was analyzed in [3] within the framework of probability theory. In this paper, the optimal threshold strategies and expected payoffs in the multi-step full-information optimal stopping game are computed for different number of players. The properties of optimal strategies are proved and results of numerical modelling are presented for both cases of full information and no information. We compare the games of different information types. The relations between the optimal thresholds in the games with full information and in that with no information are shown for two and infinite number of steps. We proved that information availability in the game changes optimal strategies of players.

This work was supported by the Institute of Applied Mathematical Research KRC of RAS and Russian Foundation for Basic Research (projects no. 16-51-55006).

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Equilibria in Altruistic Economic Growth Models

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Keywords: *Markov perfect equilibrium, altruistic growth model, intergenerational game.*

We consider a stochastic economic growth model in the form of an intergenerational dynamic game. Both paternalistic and non-paternalistic components are present in the model. Under very general assumptions allowing for unbounded utility functions and weakly continuous transitions, we establish the existence of Markov perfect equilibria that consist of a consumption strategy and an indirect utility function. In the pure paternalistic case, we obtain new results on equilibria. An important point of our contribution is that we make no separability assumptions on the utility functions of generations.



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Dynamic Models, Learning and Water Management Issues

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Keywords: *learning, water management, dynamic games.*

We consider a groundwater exploitation problem as in Rubio & Casino (2003). Water extraction is the only input in the production process of the farmers and the dynamics is given by the evolution of the level of the water table.

There are two levels of interactions: The farmers decide their own strategy (i.e., how much water they will extract from the water table) dynamically, taking into account their beliefs about the other farmer's actions. Interactions depend also on the profit function through the extraction cost that is a decreasing function of the height of the water table. Several papers solve this problem assuming complete information. They study and compare social optimum, feedback and open loop equilibrium with myopic behavior (Rubio & Casino (2003)).

In this work we consider the case where farmers have different kinds of information about other farmers' profit functions and about pumping cost functions. Farmers behave more or less myopically.

We analyze two models where we assume that each farmer relies on simple beliefs about the other farmer's behavior (Quérou & Tidball (2010)). In a first model, farmers assume that a variation of their own extraction has a first order linear effect on the extractions of others. In a second model, farmers consider that extraction of the others players is a proportion of the available water. We use a simple learning procedure where farmers' beliefs are updated through observations of the resource level over time. We also consider two other models without learning. In the first one, farmers do not know the profit function of the other farmers and cost is announced before extraction. This is the usual case of myopia. In the second one farmers know the profit function of the other player and cost is announced after extraction. In this last case farmers play a Nash equilibrium. We called this case strategic myopia. We compare the four behaviors from the economic and environmental point of view.

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Expected Utility Preferences versus Prospect Theory Preferences in Bargaining

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Keywords: *bargaining, expected utility, prospect theory, reference point dependence, loss aversion, probability weighting.*

Are individuals always better off with expected utility preferences than with non-expected utility preferences? In context of a bargaining game, I present a necessary and sufficient condition for individuals to be better off with prospect theory preferences than with expected utility preferences. I decompose the constituting elements of prospect theory to show that loss-aversion does not have an effect on the bargaining outcome, reference-point dependence is unambiguously advantageous and probability weighting is unambiguously disadvantageous. This explains the main result: individuals are better off with prospect theory preferences if and only if the effect of reference-point dependence is stronger than that of probability weighting.



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Does Cross-Franchising effects Regional Development? Evidences from the Emerging Consumer Markets

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Keywords: *trade Sales, promotion strategies, marketing performance, trade contest.*

The elevation of sales and marketing performances are dependent upon the different sales promotional strategies. Cross-franchising or transshipment of products on the other hand, is a severe concern as it sabotages regional development of the consumer markets. The transshipment of products from one region to another greatly affects sales and marketing performances of the infiltrated region. The respective study tends to unfold different dimensions of trade sales promotions and measures dimensionally its effects on marketing performance. Since cross-franchising is anticipated to shake the entire working canvas of sales; therefore its moderating effect is being measured. Sample size of 258 respondents using survey questionnaires were distributed among the retailers and distributors of FMCG products sellers of Pakistan. Structural Equation Modelling (SEM) on MPlus software was conducted in order to determine causal relationships among the constructs. The statistical results attained affirmed the established theoretical foundations.



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An Example of a Hybrid Two-Stage Dynamic Control System with Three Participants

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Keywords: *optimal control problem, non-zero differential game, hybrid dynamic control system.*

The equations of motion of a controlled system in the two-step problem under consideration for a fixed period of time contain controls from one or two or three players. At the first stage of the controlled process (from the initial moment to a predetermined moment), only the first player controls the system, which solves the problem of optimal control with a given terminal functional. In the second stage of the process, the first player decides whether the second player, or the third player, or both will participate in the control process for the remaining time or not. Supposed to that for participation, incoming players must pay a fixed amount to the first player. If “yes”, then a non-antagonistic positional differential game is played, in which Nash equilibrium is taken as a solution. If “no”, then until the end of the process, the first player continues solve the problem of optimal control.



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WORLD SCIENTIFIC



Comparative Analysis of Strategies of Stock Exchange Players Based on Game Theory

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Keywords: *game theory, economics, stock exchange players.*

In neoclassical economics, a rational agent resides within a specific system of institutions, including property rights, monetary relations, and market competition. Each agent perceives such an “environment” as a given and takes into account to maximize the benefit. Purely mathematically, this means the following: choosing directions and identifying ways of activity that will make it possible in these circumstances to maximize the benefit. Rationality of behavior means the subordination of one goal. In this case, the choice can be represented as a mathematical problem that can be solved within the framework of game theory — when the choice depends not only on the strategy of one agent but also on the market environment and strategies chosen by other agents. Systematic study of the conflicts of players and their strategies is quite a difficult task. Any generalizations are limited by the complexity of the financial market, the challenges facing its participants, and the requirements of the researcher. It is quite convenient to describe the randomness of transactions - successes and failures in the form of probabilities of transitions from one stock exchange state to another. When moving to a new state, the system no longer takes into account the circumstances of how it got into this state. Such development is described by the Markov process, that is, for each time t , the probability of any state of the exchange in the future depends only on its present state and does not depend on the past — on how the exchange came to this state, that is, the process is considered without consequences. The transition to a new state occurs with each new act of sale and purchase - a sequence of random transactions corresponds to the transition states of the system. Since the exchange system changes randomly, it is impossible to predict with certainty its condition in the future. However, the statistical properties of the future system can be predicted. In many applications, it is these statistical properties that are important. The famous Markov chain allows you to make a random «walk» along the numerical line, where at each step the position can change by +1 or -1 with equal probability. From any position two transitions to the next or previous state are possible. Transition probabilities depend only on the current position, and not on the way this position was achieved. The author reveals how the technology of game theory offers new ideas for the organization and management of exchange systems and comparative analysis of strategies.

Implementation with Secrets

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Keywords: *mechanism design, privacy preservation, strategically interdependent valuations, dominant-strategy incentive compatibility, characterization, non-cooperative games, cooperative games.*

I present a new approach — implementation with secrets — to handle situations where agents have strategically interdependent types. I demonstrate that such situations can arise in (e.g.) entry auctions for markets, and I consider the possibility that types depend on cooperative or non-cooperative strategic behavior.

I explore a class of mechanisms that allow agents to have secret information, which I define as private information that is not reported to the mechanism designer. It turns out that secrets provide a straightforward way of dealing with strategically interdependent types. My main contribution is characterizing dominant strategy incentive compatibility of mechanisms that are robust to secrets in cooperative and non-cooperative environments. The model allows agents to have multi-dimensional types and quasi-linear preferences.



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Stochastic Analogue of Cournot Model

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Keywords: *Cournot model, Nash equilibrium, stochastic programming, chance constrained optimization problem.*

The present study is concerned with a Cournot model with random demand function. Parameters of this function are considered to be normal random variables with defined mean values and standard deviations. We propose a methodology of finding Nash equilibrium under uncertainty conditions with respect to the demand function. Such methodology includes a formulation of chance constrained optimization problem within stochastic programming theory. The equilibrium of the stochastic Cournot model is attained with a certain level of probability defined beforehand. The results of numerical experiment for stochastic version of the Cournot model will be given in this paper.



Journals in Game Theory

DYNAMIC GAMES AND APPLICATIONS

Editor-in-Chief
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Birkhäuser, Boston



Penalty Method for Games of Constraints

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Keywords: *game of constraints, composite system performance, zero-sum games, penalty method.*

We consider a general problem of performance evaluation for some composite system involving subsystems. This system can produce several kinds of commodities having different load volumes, so that each kind of commodity (or each kind of work) may be accomplished with either some fixed collection of the subsystems or several alternative fixed collections of the subsystems with some pre-defined order and level of load. The subsystems capacity may be changed under influence of some activity both for restriction of their performance and for protection from this restriction. For a given profile of interference/protection levels the problem of performance evaluation for the whole composite system is determined as the optimization problem. In such a way, we obtain an "attack-defense" type antagonistic game where calculation of the value of the utility function requires a solution of the optimization problem.

In the talk we propose to modify the formulation of the above game problem in order to evaluate guaranteed system performance in the general case. The problem suggested to be solved by a penalty optimization method. This enables one to essentially simplify the solution process in comparison with finding worst case strategies in the custom zero-sum game and to create rather simple solution methods. Some examples of applications are also described.



МАТЕМАТИЧЕСКАЯ ТЕОРИЯ ИГР И ЕЕ ПРИЛОЖЕНИЯ

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Methods of Game Theory in Proxy War Studies

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Keywords: *game theory, proxy conflicts, proxy wars, principals, agents*

A typical for the last decades has become such phenomena as proxy conflicts (or proxy wars). This concept is traditionally understood as such forms of conflict, in which don't participate the immediate entertainer (principals), but their representatives (or agents). Of course, world history has a huge number of examples in which the opposing sides tried to act «by someone else's hands». However, it was precisely the last half of the 20th century that was marked by the global prevalence of proxy wars over traditional «clear» (direct) wars. Studies of the causes of such phenomena are undoubtedly interesting, since their results allow us to broaden the range of our ideas about the nature of the modern world political and economic system, and also to predict the potential trajectories of its evolution. For a mathematical description of the problems was proposed the next functional dependences

$$u_i(x_i, x_j, y_i, y_j) = \frac{x_i - y_i}{x_i - y_j + h} + c_i \frac{y_i}{y_j + h} - a_i(x_i + x_j - y_i - y_j)^{1+\alpha} - b_i(y_i + y_j)^{1+\beta},$$

x_i — level («depth») of involving players-principals in conflict;

y_i — level («depth») of involving in conflict for the agent of the principal i ;

a_i — coefficient reflecting the damage, depending on the selected levels of the direct confrontation between principals (for player i);

$\alpha > 0$ — parameter, which are modelling the effect non-linear growth of damage in the expansion of the scale of conflict;

b_i — transformation coefficient («price») of expenditures («investing») in agent;

c_i — transformation coefficient reflecting the damage, depending on the selected levels of the confrontation on the agents level (for player i);

$\beta > 0$ — parameter, which are modelling the effect non-linear growth of damage in the expansion of the scale of conflict for agents level (analogue of the coeff. α).

The researches of Nash equilibrium situations, in particular, allow one to find explanations for the objective appropriateness (necessity) of involving agent players in a conflict and, accordingly, transforming it into a proxy conflict. The main directions of development of the proposed models are:

- complicating the scheme of relationships between principals and agents, considering asymmetric schemes (for example, «two principals - three agents»);
- consideration of the problems of the ambiguity in the «principal-agent bundle» (one and the same participant under different conditions may be an agent of different principals, the problem of the unprincipled ally or so-called «myrmidon problem»);
- the problem of agent-principal feedback (the so-called problem of North Korea, the study of the mechanisms of dependence of principals on agents);
- researches of the possibilities of using cooperative game-theoretic approaches for modeling possible coalition configurations between agents and principals).

Nash Bargaining Solution as Negotiation Concept for Resource Allocation Problem — Analysis of Experimental Data

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Keywords: *resources allocation problem, Nash bargaining solution, Nash implementation*

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Motivated by research works on Zeuthen-Hicks bargaining which leads to Nash bargaining solution (Harsanyi, 1956, Vetschera, 2018) we conduct analysis of data obtained during experimental gaming comparison of resource allocation rules in case of transferable utilities described in Korgin & Korepanov 2017. In that research several mechanisms were compared in setting with quasi-linear utilities: mechanism (YH) from the class of proportional allocation mechanisms (see Başar & Maheswaran, 2003 and Yang & Hajek, 2005), mechanism with balanced payments (GL) introduced in Korgin 2013 using the Groves–Ledyard rule (Groves & Ledyard, 1977) that gives the efficient solution of the problem as a Nash equilibrium in the agents’ game, as well as its modification (GLR) reducing the dimension of the action space of the agents (Korgin & Korepanov 2016) and mechanism on the basis of distributed optimization algorithm (ADMM) problem (Boyd, Parikh & Chu, 2011). Analysis conducted over decisions made by participants in games shows that significant share of all decisions leads to increase of Nash bargaining value, and it is even higher than share of decisions, which are in agreement with best-response concept. We discuss how this decision corresponds with other type of behavior actively exhibited by participants of these experiments – so-called constant behavior and finish of negotiation process in games conducted.

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Assessment of Nash and Pareto Solutions in Three-Dimensional Auction Game

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Keywords: *dynamic games, Nash and Pareto solutions, equilibrium search algorithms*

In the paper an approach for dynamic games is developed for the search of maximum Pareto points which possess special decomposition characteristics, that inherits the properties of competitive Nash equilibrium. We consider the prototype of the model on carbon dioxide emissions reduction in the atmosphere, in which coefficients of transboundary transfer are the same for all players. It is assumed that utility functions of players consist of two components: logarithmic ecological benefit function and quadratic cost function with heterogeneous coefficients for the countries participating in the auction. Formulas are derived for competitive Nash equilibrium in a dynamic auction game. The set of Pareto points is constructed for cooperative actions of players. On the set of Pareto there stands out a point of market equilibrium and the problem is posed to shift players from competitive Nash equilibrium to this point of market equilibrium. For this shift it is required to construct an algorithm of auction type. An algorithm includes analytical calculation of the players' best replies to auction prices. It is worth to note, that surfaces of best replies connect the point of competitive Nash equilibrium with the point of market equilibrium, which lies on the set of cooperative Pareto solutions. The algorithm is implemented as a system of differential equations, right sides of which describes the mechanism of formation of auction prices and players' best replies to these prices. Trajectories of this dynamic system describe the shift of players from Nash equilibrium to Pareto maximum and show stable convergence of the algorithm. The results of the algorithm are demonstrated by the model of the auction game of fast growing economies.

Analysis of Economic Behaviour in Evolutionary Model of Tax Control Under Information Diffusion

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Keywords: *SIAP model, epidemic process, networks, evolutionary game, information diffusion, tax evasions, tax control.*

In many areas of economic interactions we face with the fact that information is an essential part of the modern lifestyle. Design of an effective fiscal system is the actual trend in the mathematical modeling of economical processes, and tax control is one of the most important components in this system. We consider a model of tax control which includes an impact of rumors and official information on the decisions of economic agents. We consider an evolutionary process of spreading information over the contact network of economic agents which involves different stages of decision making process. Recent studies have shown that tax control over total population is efficient to stimulate tax collection but very expensive procedure. Thereby diffusion of information about future tax audit can be used as an auxiliary tool for tax authority which can improve tax collection. We suppose that economic agents receive information about future tax audit from different sources: relatives and friends, media, official organizations. We also assume that economic agents make decision about their style of behavior (honest, evaders) according to the authenticity of information. In this model we also take into account agents propensity to risk, which means that taxpayer can be risk-neutral, risk-loving and risk-averse. Agents in population are also able to share information over their own contact network and then the process of information spreading resembles an epidemic process and changes the behavior of economic agents. We also show that the decision-making mechanisms of agents on the network can be different and depend largely on the psychological characteristics of each agent (propensity to risk), his/her confidence level to the information received, and, finally, the information he/she receives by analyzing the behavior of other agents. Therefore, to analyze the behavior of agents in the network, we did not only take into account the different propensity to risk of agents, but also considered various dynamics which based on the rule of proportional imitation, the rule of the most significant neighbor who has been chosen by the number of connections or by the greatest profit function. In our work we formulate a mathematical model to define a process of propagation of information and take into account different levels of contact networks such as network of close relatives and friends, social networks, etc. For this framework we analyze behavior of economic agents depending of received information and estimate aggregated taxes in the cases of presence and absence of circulated information about possible tax audit in population of economic agents. We support our model by series of numerical experiments and estimate main behavioral trends.

Behavior Models Research on the Limited Resource Allocation in Business Games on the Player Type Depending

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Keywords: *business game, utility function, resource allocation mechanisms, behavioral model, research.*

This paper describes a strategic behavior players model of limited resource allocation in business games which used different mechanisms with penalty functions. The graphical method carried out clustering by groups. It is revealed that there are 3 players' types for any allocation mechanism with penalty. Each group differs in characteristics. The factors that affect the result are investigated. We used results of games with mechanisms: Alternating direction method of multipliers (ADMM), Groves–Ledyard (GL), Groves–Ledyard reduced (GLR), Uniform mechanism (UNI), Yang and Hajek (YH).

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Periodicals in Game Theory

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A-subgame Consistent Solutions for Multicriteria Multistage Games

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Keywords: *multicriteria games, multistage games, Pareto equilibria, Shapley value, time consistency.*

The paper focuses on dynamical properties (efficiency, time consistency, dynamical compatibility – see, e.g., [2, 3]) of non-cooperative and cooperative solutions in multicriteria multistage games in extensive form with perfect information [1,2]. We exploit the A-subgame concept [1], that takes into account only the "active" players, i.e. the players which have at least one decision node in this subgame. The set of all (strong) Pareto equilibria is proved to satisfy time consistency but does not satisfy the dynamical compatibility condition [1,4]. The first step of cooperation in dynamic multicriteria game implies choosing an optimal cooperative trajectory which guarantees the largest (Pareto optimal) total vector payoff. Using an illustrative example we prove that to design time consistent cooperative agreement one need to take into account the payoffs of all the players (not only the active players). However the players can use the concept of A-subgame later, when constructing the appropriate imputation distribution procedure (IDP) [1, 2] that ensures the sustainability of a long-term cooperative agreement. In the paper we assume that the players employ the refined leximin algorithm, introduced in [2], to select a unique Pareto optimal solution for each multicriteria optimization problem they face. This approach allows to construct time consistent cooperative trajectory and vector-valued characteristic function. When designing an imputation distribution procedure for the Shapley value we assume that every active player should receive nonzero payment right after her move while the other players should get zero current payments at corresponding node. We introduce the A-incremental IDP which satisfies this so-called RIM (Reward Immediately after the Move) assumption [4], efficiency and time consistency. We consider the implementation of the A-incremental imputation distribution procedure for given multicriteria multistage game.

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Sustainable Cooperation in Multistage Multicriteria Games

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Keywords: *multicriteria games, multistage games, Pareto equilibria, Shapley value, time consistency.*

This paper deals with the dynamical properties of cooperative behavior in n -person multicriteria multistage games with perfect information [2, 3]. In order to achieve and implement a long-term cooperative agreement in multicriteria dynamic games we have to solve the following problems. First, when players seek to achieve the maximal total vector payoff of the grand coalition, they face the problem of choosing a unique Pareto optimal payoffs vector. In the dynamic setting it is necessary that a specific method the players agreed to accept in order to select a particular Pareto optimal solution satisfies time consistency [1, 2], i.e., a fragment of the optimal cooperative trajectory in the subgame should remain optimal in this subgame. We use the rule of minimal sum of relative deviations (MSRD) [3] from the ideal payoffs vector to find a unique optimal cooperative trajectory, which is proved to satisfy time consistency.

After selecting the cooperative trajectory it is necessary to construct a vector-valued characteristic function. To this end, we use the ζ -characteristic function introduced in [2] and the MSRD rule in order to select a particular Pareto efficient solution for the auxiliary vector optimization problems.

Lastly, to guarantee the sustainability of the achieved long-term cooperative agreement one needs to design a consistent imputation distribution procedure (IDP) (see, e.g., [1, 2, 3]) that should satisfy a set of useful properties. We establish the exact set of properties that an IDP has to satisfy in order to guarantee the strong time consistency of the core.

The research was funded by RFBR under the research project 18-00-00727 (18-00-00725).

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Advertising in Social Networks as a Stackelberg Game

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Keywords: *social networks, marketing, advertisement campaign, Stackelberg game, privacy.*

In recent years the social media penetration in our daily lives increases continuously and the advertisement of a new product, as well as the propagation of new ideas, the raise of social awareness on important issues or the propaganda of a political party are significantly oriented to the social media platforms. In this work, we will refer to all the aforementioned activities with the term marketing. A lot of new challenges arise when marketing is applied in social media, such as the opportunity for enhancing the indirect or implicit marketing, where the user is engaged in the procedure of evaluation and propagation of the advertisement. Based on this strategy, a well studied technique of marketing campaigns is to focus on the most influential users, who are assumed to be persuaded, or paid if needed, to adopt a positive attitude for the product or the idea advertised and spread it through their social connections to the rest of the social network.

However, the great technological progress in the field of data mining and big data analysis, alongside with the continuously increasing amount of personal information that the users provide voluntarily to the social networks, enable the marketers to perceive the preferences of many more users and to expand considerably their target set - even to address to each user personally. Nevertheless, there still exist limitations for the marketers. At first, the resources for a campaign are constrained, so the marketer has no sufficient budget to bribe many users and mainly to purchase information about the users' personal data from the social network's provider. Moreover, the access to personal data and the content relevancy of an advertisement for specific audiences are subject to legal restrictions. Finally, the users, who have the active role of evaluating and spreading an advertisement, are rational and they have their own criteria which determine their attitude. These criteria depend on their perceived benefits, which are related to the advertisement's content and to their peer pressure, as well as on their perceived costs, which are related primarily to their privacy concerns.

We formulate the interaction between the marketer and the social network's users during an advertising campaign as a Stackelberg game on a graph, where the marketer is the leader and the users are the followers. The marketer tries to maximize the average receptivity of the advertisement, throughout estimating the preferences of each user and designing a proper message for her, while paying a cost for this procedure, which is constrained by his budget. On the other hand, the users respond to the received messages in accordance with their criteria, which depend both on their preferences about content and privacy and on their position on the graph. The marketer, thus, cares firstly to find the best trade off between personalising the content of the advertisement and not violating profoundly the user's privacy and secondly to distribute properly his effort or budget to the population of the users.

Evolutionary Dynamics of Compliance in a Two-Population Game of Auditors and Auditees

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Keywords: *compliance, tax evasion, auditing, evolutionary dynamics.*

The paper studies the dynamics of compliance in a population of agents that can decide whether to comply with a prescribed behavior or not. These agents are subject to controls by auditors who might also be honest and truthfully report a fraudulent behavior of an auditee or be corrupt and get a bribe for not reporting the unlawful behavior of the auditee. Applied to the example of tax evasion, we study the case in which the level of evasion by the auditees is obtained by expected utility maximization. We then investigate the rich dynamic scenarios that may arise through the interaction of auditors and auditees.



Journals in Game Theory

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When Does the Platform Tell You the Truth? Optimal Design of Persuasion Policy in the Two-Sided Market

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Keywords: *Bayesian persuasion, two-sided market, platform, stochastic service, guaranteed service.*

In this study, we analyze the information announcement strategy for the C2C Service platform, whereas the platform provides not only a less profitable standard service with stochastic stock-out risk, but also a more profitable guaranteed service with no stock-out risk (such as the premier service). While the expected stock-out rate of the standard service is commonly known, customers do not observe the availability of the standard service when they choose between these two services, and thus their decision can be influenced by the service availability information announced by the platform. By leveraging the stock-out forecasting technique, we show that the platform has incentive to manipulate its disclosure policy via a Bayesian persuasion approach, first introduced in Kamenica and Gentzkow (2011). This implies that the platform will not always truthfully disclose the stock-out rate of the standard service for profit maximization concern. Thus, designing a state-contingent probabilistic disclosure policy, which allows the platform to announce a higher stock-out rate of the standard service to persuade customers to adopt the guaranteed service and hence involves lying under some conditions, will be optimal for the platform to boost its profit.

We solve for the optimal Bayesian persuasion strategy for the platform, and fully characterize the conditions under which the platform tells the truth, the platform tells the partial truth (telling the truth with a probability), and the platform tells all lies (or does not provide any information), respectively. We also investigate the optimal truth telling probability of the equilibrium outcomes in the short-run and in the long-run, and conduct analyses on the consequences for the platform's profit, suppliers' profit and consumers' surplus. Our study has important policy implications and provide a better understanding about the information manipulation incentive of the platform in a two-sided market of differentiated service with information asymmetry between the platform and customers.

Dynamic Shapley Value for 2-Stage Spanning Forest Game with Perishable Products

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Keywords: *dynamic Shapley value, perishable product, dynamic game.*

In the paper, a 2-stage minimum cost spanning forest game with perishable products that has N -person and M -supplier are considered. The cooperative behavior of players is defined. Selecting strategies, players build a minimum cost spanning forest at each stage. The total cost consists of costs defined by minimal cost spanning forest and the lost values of perishable products. Before the second stage, the demand of a particular player j , which is from the set N , may change with probability p that depends on the previous behavior of players. As the optimality principle, the modified Shapley Value is proposed. Computation of the Shapley Value along different cooperative path scenarios shows its subgame inconsistency.



Journals in Game Theory

GAMES AND ECONOMIC BEHAVIOR

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Patron Networks and Project Partners

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Keywords: *social networks, project evaluation, patrons, partnership, endogenous network formation.*

In many scenarios in either industry and academia, the success of a proposal can depend upon the person evaluating it. When individuals can develop acquaintances, friendships and partnerships, how will the desire for a successful project affect link formations? We study social network structures in a game of joint project endorsement with patrons. Players form links with one another under the possibility of supporting one another in project endeavors. Each player can also form a project partnership from among their links. When the partnership is submitted for evaluation, any individual in the extended network may be asked to serve as evaluator. We show that in the case of undirected link formation, any pairwise stable friendship network is a complete bipartite network, while the star network provides maximum efficiency. We further establish the stochastically stable equilibrium under mutations in link formations, and derive further results under the case of directed link formation.



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GAME THEORY AND APPLICATIONS

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Equilibria in Pure Stationary Strategies for Discounted Stochastic Positional Games

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Keywords: *stochastic positional game, discounted payoff, stationary strategy, pure stationary Nash equilibrium.*

A discounted stochastic positional game is determined by the following elements (see [1,2]): a state space X (which we assume to be finite); a partition $X = X_1 \cup X_2 \cup \dots \cup X_m$, where X_i represents the position set of player $i \in \{1, 2, \dots, m\}$; a finite set $A(x)$ of actions in each state $x \in X$; a step reward $f^i(x, a)$ with respect to each player $i \in \{1, 2, \dots, m\}$ in each state $x \in X$ and for an arbitrary action $a \in A(x)$; a transition probability function $p : X \times \prod_{x \in X} A(x) \times X \rightarrow [0, 1]$ that gives the probability transitions $p_{x,y}^a$ from an arbitrary $x \in X$ to an arbitrary $y \in Y$ for a fixed action $a \in A(x)$, where $\sum_{y \in X} p_{x,y}^a = 1, \forall x \in X, a \in A(x)$; a discount factor $\lambda, 0 < \lambda < 1$, and a starting state $x_0 \in X$.

The game starts at the moment of time $t = 0$ in the state x_0 where the player $i \in \{1, 2, \dots, m\}$ who is the owner of the state position x_0 ($x_0 \in X_i$) chooses an action $a_0 \in A(x_0)$ and determines the rewards $f^1(x_0, a_0), f^2(x_0, a_0), \dots, f^m(x_0, a_0)$ for the corresponding players $1, 2, \dots, m$. After that the game passes to a state $y = x_1 \in X$ according to probability distribution $\{p_{x_0,y}^{a_0}\}$. At the moment of time $t = 1$ the player $k \in \{1, 2, \dots, m\}$ who is the owner of the state position x_1 ($x_1 \in X_k$) chooses an action $a_1 \in A(x_1)$ and players $1, 2, \dots, m$ receive the corresponding rewards $f^1(x_1, a_1), f^2(x_1, a_1), \dots, f^m(x_1, a_1)$. Then the game passes to a state $y = x_2 \in X$ according to probability distribution $\{p_{x_1,y}^{a_1}\}$ and so on indefinitely. Such a play of the game produces a sequence of states and actions $x_0, a_0, x_1, a_1, \dots, x_t, a_t, \dots$ that defines a stream of stage rewards $f^1(x_t, a_t), f^2(x_t, a_t), \dots, f^m(x_t, a_t), t = 0, 1, 2, \dots$. The *discounted stochastic positional game* is the game with payoffs of the players $F_{x_0}^i = \mathbb{E}(\sum_{\tau=0}^{\infty} \lambda^\tau f^i(x_\tau, a_\tau))$, $i = 1, 2, \dots, m$, where each player intends to maximize the expected discounted sum of his stage rewards. We consider discounted stochastic positional games when players use stationary strategies of choosing the actions in the states. A *mixed stationary strategy* of player i is a mapping s^i that provides for every $x \in X_i$ a probability distribution over $A(x)$. So, a stationary strategy of player i is given if for each $x \in X_i$ and an arbitrary $a \in A(x)$ are given $s_{x,a}^i \geq 0, \forall a \in A(x)$ such that $\sum_{a \in A(x)} s_{x,a}^i = 1$. If these probabilities take only values 0 and 1 then we have a pure stationary strategy. We show that an arbitrary discounted stochastic game possesses Nash equilibria in pure stationary strategies.

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Partial Sum of Matrix Games

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Keywords: *matrix game partial, sum of games, linear programming, statistical games.*

In game theory, matrix games are an important and well-designed class of games. An important task for such games is to find their solutions in mixed strategies. For this purpose, as a rule, linear programming problems are built, the solutions of which are optimal mixed strategies of players and the value of the matrix game. Unfortunately, not all such problems are easy to solve, as their dimension is very large. A similar problem occurs, for example, when solving statistical games or «agent against the center» games.

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Competition in Defensive and Offensive Advertising Strategies in a Segmented Market

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Keywords: *game theory, goodwill, market segmentation, dual dynamic approach, numerical algorithm.*

Modern technologies allow marketers to adapt marketing messages to the individual preferences of each customer. In response to these practices, there is a requirement to create a tool that allows marketers to determine optimal advertising strategies for use in a market that is divided into infinite segments.

This paper develops the idea of modelling competition in a market with infinite segments. Namely, a partial differential game of goodwill dynamics is presented that incorporates some new realistic features. Both companies manage two types of advertising strategies that are designed to increase goodwill: defensive and offensive. We assume that companies compete for potential consumers through offensive advertising which is directed only at this part of the market. The strength of the competition depends on the level of the product homogeneity. On the other hand, the defensive strategies are focused on the maintenance of the existing customers and are tailored to each consumer. Thus, we are considering personalized advertising activities.

For the partial differential game, we propose, for the first time, the sufficient condition for the existence of the open-loop Nash advertising equilibrium using the dual approach. Moreover, based on the concept of ε -open-loop equilibrium, we propose the use of a numerical scheme to obtain an optimal solution in the finite steps. The proposed numerical procedure allow us to investigate how the homogeneity of competing products and customer recommendations affect advertising and goodwill.

Political Equilibrium Social Security with Migration

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Keywords: *social security, voting, coalition formation, legislative bargaining, taxation, overlapping generations.*

We analyze the consequences on social security of immigration in a micro founded parliamentary democracy (with voting, endogenous coalition formation, legislative bargaining). The underlying economic model is an overlapping-generations economy, where individuals differ in preferences over a public good (as well as in age). Social security, as well as the public good is funded by labour- and capital taxes. We find that in an economy with positive population growth, immigration increases the social security payment (per old). On the contrary, an economy with (small) population decline will experience a decline in social security if immigration is large enough. The reason is that immigration in the latter case alters the political majority. Common demographic transitions in OECD countries have caused falls in both birth rates and death rates, in turn implying a higher ratio of old to young (an "ageing" population). Recent debate has centred on the consequences for pay-as-you-go pension systems. As these systems rely on taxing the young and transfer to the old, they are only sustainable if there is population growth. It has been argued in the policy debate, as well as in academic papers, such as Blake and Mayhew (2006), that immigration may be one solution. Immigration of young workers alters the ratio of old to young and works as a (temporary) increase in the population growth rate.

This argument relies on the assumption that the social security tax remains the same. However, one would not expect the social security tax to be invariant with respect to the demographic structure. On one hand, an increase in the size of the young brings a resource gain, as the young bring a labour endowment that can be taxed and redistributed. This resource gain can, at least partially, be redistributed to the old in form of increased social security. On the other hand, an increase in the size of the young may shift the political powers to the young and since the young have no interest in social security (as long as they are young) the political-equilibrium level of social security may be reduced. Razin and Sand (2007) examine political equilibrium social security in an overlapping generations model. Their decisive voter is either young or old, implying corners in equilibrium. When the decisive voter is old, society pays the maximum amount of social security, and when the decisive voter is young there is no social security. Gonzalez-Eiras and Niepelt (2007) use a probabilistic voting model, implying a government objective equivalent to a social welfare function. This formulation avoids the corners of no or maximum social security. Their result is that an increase in immigration raises social security payments. The reason is that a utilitarian social welfare function attains a higher value when the resource gain from immigration is spread across both young and old (and not just the young).

Privately Provided Public Goods in a Dynamic Economy with Heterogeneous Individuals

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Keywords: *differential games, public goods, private provision.*

We analyse privately provided public goods in a dynamic economy where individuals accumulate capital over time and where the distributions of income are endogenous. We characterize equilibria both when the individuals can commit to their future contributions (open-loop equilibria) and when they cannot (feedback equilibria). In steady state, larger economies have more contributors. If the public good is normal, then its quantity increases in population size in the open-loop equilibrium, but not necessarily in the feedback equilibrium. If both private and public goods are normal, then the open-loop equilibrium exhibits greater steady-state public provision than the feedback equilibrium. If private consumption is inferior the opposite is true. We show that over time income distributions converge and all individuals become contributors.



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Reclamation of an Extraction Site

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Keywords: *resource extraction, land reclamation, differential games.*

We study a resource extraction site reclamation problem in a differential game setting in which firms' reclamation cost at the end of their license duration depends on the damage their activities cause to a common environment. We analyze and compare individual firms' choices and extent of disturbed environment under non-cooperative and cooperative assumptions, by distinguishing between situations in which firms are homogeneous and heterogeneous. Heterogeneity allows us to study the situation in which firms have different license durations and its consequences on individual firms' behavior and environmental outcomes.

Extraction activities severely deteriorate the environment on and near the extraction field in several ways. Reclamation and rehabilitation of the extraction sites are crucial in order to ensure that the environment remains viable for the use of future generations. Despite the clear importance of mine reclamation which has become even more crucial with the increasing number of extraction sites all over the world due to both increase in demand and improvement in extraction technologies, the issue has rarely been discussed in the literature. Our paper tries to make a first theoretical contribution on this topic when firms' decisions and environmental outcomes are impacted by the presence of reclamation costs. We develop a theoretical model of extraction site reclamation from the point of view of two neighboring firms which plan for their reclamation activities strategically. We assume that the firms are liable to clean-up at the end of their extraction license, and that the reclamation cost depends on the extent of the environmental damage associated with their joint extraction activities during the entire license duration. We analyze and compare how individual firms' behavior and environmental outcomes change under noncooperation and cooperation scenarios, by distinguishing between situations in which firms are homogeneous and heterogeneous. To the best of our knowledge, this is the first paper analyzing the implication of reclamation costs on firms' behavior and environmental outcomes, and thus we have maintained our framework as simple as possible. However, this has precluded us to endogenize extraction choices. This further task is left for future research.

Game Equilibria and Unification Dynamics in Networks with Homogenous Agents and Knowledge Externalities

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Keywords: *network, network game, Nash equilibrium, externality, productivity, innovation cluster.*

In this paper, we consider the following problem - what affects the Nash equilibrium amount of investment in knowledge when one of the complete graph enters another full one. The solution of this problem will allow us to understand exactly how game agents will behave when deciding whether to enter the other net, what conditions and externalities affect it and how the level of future equilibrium amount of investments in knowledge can be predicted.



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Evolutionary Game Theory Control for Consensus

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Keywords: *learning algorithms, potential game, evolutionary game theory, population dynamics, robotics.*

Several techniques are relevant for formation flying in swarming applications. Formation flying often means reaching an agreement, i.e. a consensus about heading, location, range spacing. The communication network topology plus controllers based on Laplacian matrices allow to define negative Laplacian flows, i.e. dynamic system evolutions that converge to prescribed network configuration (to minimize the network energy, to minimize disagreements in networks between nodes / agents / players). Laplacian matrices are also tools that allow creation of potential game cost matrices, cost matrices that satisfy the convergence properties of potential games. Then, learning algorithms from game theory can be applied to reach equilibrium states. The main advantage to use game theory learning algorithms rather than Laplacian controllers is that the Laplacian matrices lead to averaging algorithms with fix weights while the game theory approach not. We will mainly focus on the evolutionary game replicator equation as learning algorithm. We will use the well-known bug problem as an illustrative example. The so-called bug problem kindly refers to robot spacing (N robots) along a circle using swarming techniques (N large, local communication only). Classical approaches lead to cyclic pursuit and balancing controls with fix gains mitigating distances respect to the previous and to the next robot («bug») on the circle. The game theory learning algorithms select decisions / controls in a different manner that can be re-interpreted as varying cyclic pursuit / balancing gains, i.e. as adaptive control gains.

Search Strategies and Comparison of the Detection Time by Team of Pursuers Evader Player on the Plane

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Keywords: *differential search game, information set, team of pursuers.*

We consider a differential simple search game without phase constraints on the plane. At the initial moment the team of pursuers $\bar{P} = \{P_1, \dots, P_k\}$ knows that the initial state of the evader belongs to the disk of radius r centered at the origin. Henceforth, they does not receive any current information about the disposition of the opponent.

Each of the pursuer player from $\bar{P} = \{P_1, \dots, P_k\}$ team chooses initial position and piecewise constant strategy. As an example, the games a simple search on the plane is considered and the necessary conditions for detection is found. For cases with two and three pursuers, comparison of the detection time of the evader E was made and numerically constructed an approximate information sets.



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On Computing Generalized Nash Equilibria in Nonconvex Quadratic Games

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Keywords: *generalized Nash equilibrium problem, Nikaido-Isoda function, support function, nonconvex optimization.*

We consider an n-player game with shared constraints, where every payoff function is a nonconvex quadratic function. The set of feasible game profiles is assumed to be convex and compact. Since we do not impose the commonly used assumption of player-convexity on payoffs, the game may have no equilibrium points. Our focus is to find the so-called normalized Nash equilibrium, which is a special kind of generalized equilibria. To this end, we suggest to use the Nikaido-Isoda function in order to reformulate the game as a minmax problem, namely as an optimization problem with implicit objective function. This function may be underestimated by explicit nonlinear support minorants. It leads to a global optimization procedure that either converges to a normalized equilibrium point or establishes that there are no such points in the game. The results of a computational experiment are presented.



Journals in Game Theory

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Allocation Problems for Several Public Resources

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Keywords: *lexicographically minmax solutions, Wardrop solution, envy free solution.*

We consider problems of "fair" distribution of several different public resources. If τ is a partition of a finite set N , each resource c_j is distributed between points of $B_j \in \tau$. We suppose that either all resources are goods or all resources are bads. There are finite projects, each project use points from its subset of N (its coalition). \mathcal{A} is the set of such coalitions. We suppose that coalitions of different projects are different. Coalitions in \mathcal{A} can intersect. The gain/loss function G_S of a project that use $S \in \mathcal{A}$ at an allocation x depends only on the restriction of the allocation on the coalition of the project, i.e., $G_S(x) = G_S(x_S)$. We consider the following 4 solutions. If all resources are goods, the lexicographically maxmin solution with respect to the set of the gain functions $\{G_S\}_{S \in \mathcal{A}}$ is defined. It generalizes the restricted nucleolus for $G_S(x) = x(S) - v(S)$. If all resources are bads, the lexicographically minmax solution with respect to the set of the loss functions is defined. It generalizes the restricted antinucleolus. A generalization of Wardrop solution is also considered for $G_S(x) = g_S(x(S))$. The fourth solution depends on an undirected graph Γ , where \mathcal{A} is the set of its nodes. The projects compare their gains/losses at fixed allocation if their coalitions are adjacent in Γ . For fixed gain/loss functions, an allocation is envy stable (or envy free) with respect to Γ if for each pair of coalitions that are adjacent in Γ , either their gains are equal or the coalition with greater gain gets 0 at the allocation. We describe conditions on \mathcal{A} , τ , and Γ that ensure the existence of envy stable solutions for each collection $\{G_S\}_{S \in \mathcal{A}}$ of continuous functions that are strictly increasing in each variable. For collections of functions $\{G_S\}_{S \in \mathcal{A}}$ such that $G_S(x_S) = g_S(\{x(S \cap B)\}_{B \in \tau})$, where g_S are continuous strictly increasing in each variable functions, the condition for inclusion of lexicographically maxmin solution and lexicographically minmax solutions in envy stable solution are obtained. A condition on \mathcal{A} , τ , and Γ that ensure inclusion of Wardrop solution in envy stable solution is described. We also describe conditions for coincidence the first three solutions. Special cases of the problem were considered at GTM2010, GTM2011, GTM2012, GTM2018.

Distribution Network Coordination with Revenue-Sharing Contract

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Keywords: *distribution supply network, multi-echelon network game, supply chain coordination, coordinating contract, revenue-sharing contract.*

Channel coordination is one of the central concepts of supply chain management. The most common definition of the supply chain coordination was given by Malone and Crowston in 1994. It was defined as the process of managing dependencies among activities (Malone and Crowston, 1994). Among different coordination mechanisms which can be implemented in supply networks, coordinating contract enables to align supply network members' objectives (Cachon, 2003), maintaining bilateral agreement and decentralized decision-making conditions, and motivates companies to cooperate for achieving results close to centralized network. The idea of the research is to explore a distribution network where agents are connected in some network relationship under revenue-sharing contract. The latter one was explored in many research (Pasternack, 1985; Cachon, 2003; Cachon and Lariviere, 2005; Dana and Spier, 2001), but only few of them investigated multi-echelon supply networks. Most existed models consist one upstream and one or multi downstream companies, named manufacturer (or supplier) and retailer or retailers. The study is aimed to show the role of contracting mechanism in achieving coordination in distribution networks and investigates cooperative multi-echelon game involving manufacturer, distributor and retailers. For the purpose of the research the contract is said to coordinate supply network if the individual and collective rationality properties are fulfilled. This means that such a contract must be a Nash equilibrium in the considering model, and possesses a Pareto-optimality property (Cachon, 2003). The game solution is the optimal parameters of the contract in terms of network coordination. The multi-echelon network model with revenue-sharing contract is proposed.

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Cooperative Solutions for the Eurasian Gas Network

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Keywords: *totally positive game, fuzzy TU cooperative game, core, super-differential.*

We relate three solutions for cooperative games, the Shapley value, the nucleolus and the core. We use a calibrated model of the Eurasian gas network, developed in Hubert and Orlova (2018)¹. Hubert and Orlova (2018) study the impact of the liberalization of EU natural gas markets on the balance of power between ‘local champions’, customers, and outside producers, such as Russian Gazprom, using the Shapley Value and the nucleolus as indicators for market power. Two solution concepts yielded different results with respect to the power redistribution.

In this paper, as in Hubert and Orlova (2018), we continue to distinguish between two steps of the reform: first, the opening of access to transit pipes and second, the opening of access to distribution systems, hence customers. As a result, we have three access regimes: fragmented market before the onset of reforms, integrated market after the first step of reform, and liberalized market after implementation of the two steps of reform. As a first result, we establish that for none of the access regimes the core is ever empty, but the Shapley value never belongs to the respective core. To obtain a differentiated picture of the (in)stability of an allocation, we propose the $n\epsilon$ -core which is a generalization of the strong ϵ -core, and define three stability measures. In general, the first metric is based on the minimal costs of establishing a coalition for a given upper bound on the size of coalitions. The second metric refers to the minimal number of players, which are necessary for setting up a coalition to veto the payoff for a given costs of establishing a coalition. The third metric is a probabilistic one. It is based on the probability of picking up a deviating coalition for the given costs and for the given upper bound on the coalition size. To analyze how the instability of the Shapley value changes with liberalization, we apply the three measures to our real life model. We find that liberalization increases the degree of instability. Opening of access to pipelines increases the minimal costs of setting up a coalition that provide the stability of the Shapley value, decreases the minimal number of players in a deviating coalition and raises the probability of selecting such a coalition if we select coalitions at random.

¹Hubert and Orlova (2018). Network Access and Market Power. Energy Economics 76, 170-185

One Way to Define Characteristic Function in Stochastic Games

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Keywords: *stochastic games, dynamic games, characteristic function, subgame consistency, strongly subgame consistency.*

In the paper we consider discounted stochastic games with finite number of states. Based on a non-cooperative stochastic game, a cooperative game is constructed by defining a characteristic function. In classical cooperative game theory, the characteristic function is supposed to be given. But in some problems there is a need to construct this function if initially the non-cooperative game is defined and players have an incentive to cooperate and realize some cooperative solution in the game. There are different approaches how to define characteristic function having a non-cooperative game [1-3,5,7-9]. The new approach of constructing the characteristic function in stochastic games is proposed. It is based on defining characteristic functions for one-shot stage games [3]. The conditions guaranteeing subgame consistency and strongly subgame consistency of the cooperative solutions are obtained (see [4,6,10] to specify the problem of strongly subgame consistency of cooperative solutions).

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Dynamic Game Models of Executors Incentives in Projects with Discrete and Continuous Time

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Keywords: *hierarchical dynamic game, new products development project, learning curve effect.*

In projects for the development of new products in industrial enterprises, the effect of the learning curve is manifested, which consists in the fact that the time spent by workers on performing of repetitive production tasks is reduced. The project for the development of new products is considered as a managed hierarchical dynamic system consisting of the management of the project (center) and executors (agents). Production activity in the project for the development of new production is characterized by the diverging interests of the center and agents, which leads to a decrease in the economic efficiency of the entire production system. The dynamics of a controlled dynamic production system depends only on the actions of the agents, while the center affects the target function of the agents by choosing the material incentive function. The study formulates the dynamic task of material incentives in projects for the development of new production with discrete and continuous time. To solve the formulated dynamic problem of material incentives, the well-known principle of cost compensation was applied. The original task is divided into the task of coordinated incentives and the task of coordinated planning. The task of coordinated incentives is as follows. The center chooses a compensatory incentive system, which consists in compensating the agent's expenses in case the center chooses optimal trajectory, or there are no material payments otherwise. In practice, the compensatory material incentive system can be implemented in the form of a penalty for deviating from the planned trajectory of the center. The study showed that the task of coordinated dynamic planning is for the center to determine the optimal planned production volumes in order to minimize the labor costs of agents. Thus, the initial dynamic problem of material incentives was reduced to the optimal control problem. The dynamic task of coordinated planning was considered in two versions: with discrete and continuous time. For the numerical solution of the formulated discrete optimal control problem, the Bellman dynamic programming method was used. The problem of optimal control with continuous time was solved analytically using the Pontryagin maximum principle. The results of the numerical and analytical solutions of coordinated dynamic tasks of planning production volumes for various models of learning curves are obtained. The dynamic tasks of coordinated planning of production volumes were solved with and without discounting. As a result of the study, a condition for the optimal production volumes determining matching the interests of the center and agents was found: the optimal production volumes for any model of the learning curve at each point in time should be chosen inversely to the labor intensity of the product and directly to the discount rate. In the absence of discounting: the optimal production volumes for any model of the learning curve at each point in time should be chosen inversely to the labor intensity of the products.

Finding Location Nash Equilibrium for Firms Competing with Cournot Quantities

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Keywords: *network competitive location, Nash equilibrium, delivered quantities.*

Location choice in spatial competition often deals with models based on a two stage game. In the first stage, the competing firms select their facility locations, in the second stage, they compete for demand with the aim of profit maximization. In this paper, we deal with firms that locate multiple facilities and offer the same type of product to the customer. Competition is performed under delivered quantities that are sent from the facilities to the customers, which are located at the nodes of a transportation network. Facility location candidates are all the points on the network. It is assumed that the competing firms deliver the Cournot equilibrium quantities once the firms have their facilities established. Since such quantities depend on the facility locations of the competing firms, the two stage game is reduced to a single stage location game which is the subject of this research.

The existence of Nash equilibria for the location game has been proved for a linear price sensitive demand when firms locate multi-facilities. Although it is known that Nash equilibria exist, a few number of papers have investigated how to find such equilibria on a network location space. To our knowledge, only illustrative examples and some methods have been proposed for firms that compete with a single facility. We will present a Mixed Integer Linear Programming model to obtain the best facility locations for any firm, assuming that the other firms have their facility locations already fixed. This formulation allows us to find location Nash equilibria by the best response method when firms locate multi-facilities. The proposed formulation can be used to solve real size problems. A study for different scenarios with 1049 demand nodes and 142 nodes as location candidates is presented and conclusions are drawn from a sensitivity analysis.

On Class of Non-cooperative Differential Games with Continuous Updating

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Keywords: *differential games with continuous updating, Nash equilibrium, Hamilton-Jacobi-Bellman equation, Pontryagin's Maximum Principal.*

The work is devoted to a new class of differential games with continuous updating. It is assumed that at each time instant, players have or use information about the game defined on a closed time interval. However, as the time evolves, information about the game updates, namely, there is a continuous shift of time interval, which determines the information available to players. Information about the game is the information about motion equations and payoff functions of players. For this class of games, direct application of classical approaches to the determination of optimality principles such as Nash equilibrium is not possible. The subject of the current paper is construction of solution concept similar to Nash equilibrium for this class of differential games and corresponding optimality conditions, in particular modernized Hamilton-Jacobi-Bellman equations and Pontryagin's maximum principle. Also, the linear quadratic case of differential game model with continuous updating is considered. Linear quadratic case for this class of games is particularly important for practical problems arising in engineering of human-machine interaction. The explicit form of Nash equilibrium for linear quadratic case is presented. Also, the case of dynamic updating for linear quadratic differential game is studied and uniform convergence of Nash equilibrium strategies and corresponding trajectory for a case of continuous updating and dynamic updating is demonstrated. Till now only a class of games with dynamic updating was studied in the papers [1], [2], [3], [4], where authors laid a foundation for further study of the class of games with dynamic updating.

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A Data Driven Coordinated Replenishment Policy

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Keywords: *coordinated replenishment, dynamic programming, coalitional games.*

We study a coordinated replenishment solution. To address uncertain demand, historical demand data are used to build and calibrate a forecasting model based on linear regression and which includes seasonality and trend. Then, we formulate and solve a stochastic optimization problem which takes as input the forecasted values, and provides as output the coordinated replenishment strategies. The cost to minimize includes a major fixed cost associated with each replenishment and a minor cost associated with an item involved in the replenishment. To quantify the benefit from coordination, a coalitional game with transferable utilities is developed in which the retailers who agree on joint replenishment can be seen as a coalition. Finally, we address issues related to the stability of the coalition in response to specific allocation policies.



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Teaching Game Theory, Countering Ethical Dilemmas

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Keywords: *prisoners' dilemma, teaching game theory, ethics.*

The Prisoners' Dilemma is a near-perfect textbook example that plays out in the reality way too often. In the classic game format, the Nash equilibrium is when the prisoners fink or 'back-stab' the partner-in-crime. While it is perfectly acceptable for the prisoners (criminals) to fink and help the police, the backstabbing raises ethical dilemmas while teaching the same in a classroom setup. There are two game setups used for classroom teaching and examination respectively, both of which reward handsomely the "backstabber". In the first classroom simulation, multiple teams compete for the highest returns on business decisions taken. Their decision is in the form of Strategy X vs Strategy Y, which fetch similar yet disguised returns like that in the classic game of prisoners' dilemma. The team gets to choose in isolation first and in later rounds, get to negotiate a joint strategy. However, invariably, they renege on the negotiated strategy and, figuratively, "backstab" the team comprising their classmates. This leaves the trusting team much worse-off while rewarding the backstabbing team handsomely. In the second simulation, a group assignment is given. Hypothetically, some participants cheat on the group assignment. The individual has a choice now to remain silent and score a lower grade or to cooperate and score a better grade than before. An even better possibility might exist where the cheating takes place but one among the cheaters 'finks'. Again, a similar set of moral dilemma emerges, which is modeled with a detailed decision tree. While MBA participants have been genuinely flabbergasted at the moral depravity of the winners, a real risk is that of sending the wrong message for perpetuity – i.e., ethical misconduct (backstabbing) is an essential step for winning in the competitive setups.

Global Optimization Approach for Finding Berge Equilibrium

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Keywords: *Berge equilibrium, optimality conditions, global search, algorithm, optimization.*

We derive necessary and sufficient conditions for a point to be Berge equilibrium [1-3] in two person matrix game. Based on optimality conditions, we reduce a problem of finding Berge equilibrium to a nonconvex optimization problem. For solving the optimization problem, we propose a global search heuristic method. Some numerical results are provided.

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Journals in Game Theory

DYNAMIC GAMES AND APPLICATIONS

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Dynamic Multicriteria Games with Different Planning Horizons

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Keywords: *dynamic games, multicriteria games, Nash bargaining solution, different planning horizons.*

Consider a bicriteria dynamic game with n participants in discrete time. The players exploit a common resource and wish to optimize two different criteria. The state dynamics is in the form

$$x_{t+1} = f(x_t, u_{1t}, \dots, u_{nt}), \quad x_0 = x, \quad (2)$$

where $x_t \geq 0$ is the resource size at time $t \geq 0$, $f(x_t, u_{1t}, \dots, u_{nt})$ denotes the natural growth function, and $u_{it} \geq 0$ gives the exploitation rate of player i at time t , $i \in N = \{1, \dots, n\}$. We explore a model in which the players possess heterogeneous planning horizons. By assumption, players harvest the fish stock during m_1, \dots, m_n steps, respectively, $m_1 < \dots < m_n$ and $m_1 > m_0 = 1$. Therefore, during the time period $[m_i, m_{i+1}]$ $n - i$ players harvest the same stock, and the problem consists in evaluating their optimal strategies. The payoff functions of the players are defined by

$$J_i = \left(\begin{array}{l} J_i^1 = \sum_{k=1}^i \sum_{t=m_{k-1}}^{m_k} \delta^t g_i^1(x_{kt}, u_{kt}, \dots, u_{nt}) \\ J_i^2 = \sum_{k=1}^i \sum_{t=m_{k-1}}^{m_k} \delta^t g_i^2(x_{kt}, u_{kt}, \dots, u_{nt}) \end{array} \right), \quad i \in N, \quad (3)$$

where $g_i^j(\cdot) \geq 0$ gives the instantaneous utility, $j = 1, 2$, $i \in N$, $\delta \in (0, 1)$ denotes a common discount factor, and x_{it} , $i = 1, \dots, n$, possess the dynamics

$$x_{it+1} = f(x_{it}, u_{it}, \dots, u_{nt}), \quad x_{it}(m_{i-1}) = x_{i-1t}(m_{i-1}).$$

We design the noncooperative equilibrium in dynamic multicriteria game applying the Nash bargaining products [2]. Then, we find a multicriteria cooperative equilibrium as a solution of a Nash bargaining scheme with the multicriteria Nash equilibrium playing the role of status quo points [3]. A discrete-time game-theoretic bioresource management model (fishery problem) with different planning horizons is treated to show how the presented concepts work. The research was supported by the Russian Science Foundation, proj.no.17-11-01079.

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Coordinated Influence on the Beliefs of Social Network Members

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Keywords: *social network, influence, opinion dynamics, linear-quadratic games, cooperation, equilibrium, centrality.*

In this talk, we examine a model of the coordinated influence in a social network in which several its members, called players, can jointly influence the beliefs of other members, called agents, during a finite number of periods. The model is considered as a cooperative dynamic game. The influence of players is expressed by declaring their beliefs which are then considered and weighted by the agents to form their own beliefs. Our goal is to find the declared beliefs of players focusing only on associated costs as well as on the average deviation of agents beliefs from the desired ones. Under coordination, the total costs of players are allocated using the Shapley value. When we have no information regarding the levels of trust for agents to each other, we estimate these values by means of a centrality measure. Numerical simulation is carried out for a well-known social network of a university karate club and for a lattice often used for modeling spatial networks.



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Optimal Incentive Strategies in Stochastic Stackelberg Games

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Keywords: *Stackelberg games, incentive strategies, stochastic optimal control.*

We consider a game where manager's (leader's) aim is to maximize the gain of a large corporation by the distribution of funds between m producers (followers). To each follower the leader reports a non-negative stimulating (incentive) function $c_i(x, a)$, depending on the state of the system x (e.g., the market price of the produced good) and the actions $a = (a^1, \dots, a^m)$ of the producers (e.g., the production levels). At each stage of the game the producers select their actions a_t^i independently and get the rewards $r_i(x_t, a_t) = c_i(x_t, a_t) - g_i(x_t, a_t)$, where g_i are the production costs. The manager, or the corporation, one-stage gain equals to $f(x_t, a_t) - \sum_{i=1}^m c_i(x_t, a_t)$, where f can be regarded as the sales revenue. The stochastic "law of motion" of the state variable x_t is governed by a transition kernel q : informally, $\mathbf{P}(x_{t+1} \in B | x_t, a_t) = q(B | x_t, a_t)$.

Each player's gain is estimated over the infinite horizon with the common discount factor β . So, $\mathbf{E} \sum_{t=0}^{\infty} \beta^t (f(x_t, a_t) - \sum_{i=1}^m c_i(x_t, a_t))$ is the objective functional of the leader, and $\mathbf{E} \sum_{t=0}^{\infty} \beta^t (c_i(x_t, a_t) - g_i(x_t, a_t))$ are the objective functionals of the followers. For each tuple (c_1, \dots, c_m) the pool of producers responds by a Nash equilibrium in the corresponding discounted stochastic game. The leader performs the optimization over the functions c_i from an appropriate class. From the previous work [?, ?] it is known that it is optimal for the leader to economically motivate the followers to implement the strategies $a_t^i = \bar{u}_i(x_t)$, where $\bar{u} = (\bar{u}_1, \dots, \bar{u}_n)$ is an optimal stationary deterministic strategy in the Markov decision problem $\mathbf{E} \sum_{t=0}^{\infty} \beta^t (f(x_t, a_t) - \sum_{i=1}^m g_i(x_t, a_t)) \rightarrow \max$.

Passing to the case of multiple followers draw some technical difficulties related to the existence of a stationary Markov equilibrium. We use a general formalism of discounted stochastic games to precisely describe an ε -optimal strategy of the leader and her value function in our model, formulated as a Stackelberg game.

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Cooperative Decision Making in Cooperative Control Systems by means of Game Theory

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Keywords: *human-machine cooperation, cooperative, decision making, event-based game, war-of-attrition.*

Cooperative control systems aim at creating synergies by combining the strengths of humans (creativity) and machines (power) and hence being more economical compared to fully automated systems. This work aims at modeling cooperative control systems on a decision level going beyond approaches in the context of Haptic Shared Control. The latter model cooperative tasks with a shared, common or at least similar goal and close physical interaction between the partners e.g. by means of differential games [1]. The scope of the decision level is to handle scenarios without a common goal. Usually, the leader-follower paradigm is implemented forcing the automation into the follower's role. However, there are situations in which this paradigm is unsuitable, e.g. if the automation receives more comprehensive information about the situation than the human does. Hence, an emancipated cooperative behavior of the automation is essential. Furthermore, cooperative decision making is influenced by the cooperating but individually rational partners and the system they control. We therefore propose to treat also cooperative decision making as a game and apply game-theoretic models and approaches. However, state-of-the-art game theory models lack either the consideration of a dynamical system that is usually present in human-machine cooperation [2, 3] or the decision process itself [1]. Therefore, we propose two modeling approaches: First, we introduce an event-based game that explicitly models the communication between players and also considers the system dynamics in a complete information setting. Second, we focus on the incomplete information setting that is more realistic if humans are involved. Here we adapt the war-of-attrition model w.r.t. its cost-structure to model the system dynamics and its effects on decision making in a more abstract way.

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Unknown Rivals in Continuous Time Games with Imperfect Observation

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Keywords: *linear quadratic differential game, learning dynamics, stochastic stability.*

We study games played between unknown rivals. We first present results for two, symmetric known rivals: symmetric Markov perfect equilibria always exist; we find a parameter condition for which multiple asymmetric equilibria exist, in which one agent benefits from the other's exertions; we identify three, independent conditions for these multiple equilibria. For multiple, symmetric known rivals, the equilibrium set grows, but retains the feature from the two-agent game: one set of agents exerts less effort, benefiting from the other's exertions. Finally, we use Markov strategies to consider unknown rivals, by assuming agents can only condition on the state variable, and are unaware of the other agents in the game. With unknown rivals, agents almost certainly learn an inefficient equilibrium relative to the set of equilibria.



Periodicals in Game Theory

CONTRIBUTIONS TO GAME THEORY AND MANAGEMENT

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Antagonistic Games Which Have not Acceptable Points

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Keywords: *antagonistic game with ordered outcomes, acceptable point, critical strategy.*

Acceptable point concept is an important generalization of the equilibrium point concept. Acceptable points in general cooperative n -person games was introduced by Aumann. We consider the acceptable point concept for antagonistic games with ordered outcomes. This report is a continuation of the author's work (see Rozen, 2018). The main problem can be presented as a description of properties for games that do not have acceptable outcomes and also a finding some sufficient conditions for such a game in the class of antagonistic games with ordered outcomes.

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Journals in Game Theory

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SPRINGER



Viability of Ecological and Economical Systems in the Presence of Risk of Climatic Disasters

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Keywords: *viability theory, risk management, coviability, discrete games.*

By browsing the path from theoretical applications of Viability Theory to the study of concrete problems we will see how it has been possible to give relevant answers to a problem whose complexity precisely reflects a reality that is delicate to apprehend. Through the lens of an agro-ecological problem presented in the paper: "Agroecological transition: A viability model to assess soil restoration" and published in the Natural Resource Modeling journal in 2017, we rely on the mathematical viability theory and its extensions to dynamical games against nature to study the sustainability of agricultural systems subject to climate uncertainty. Our objective is to determine farming practices and activity sequences that allow to restore soil quality and value the natural capital and the services provided by these ecosystems in the presence of risk of major climatic disasters. This is an on-going joint work with Anya Désilles, Aïchouche Oubraham and Georges Zaccour. In this context we will see why it is necessary, when considering real problems like those inferring coviability of social and ecological Systems, to consider discrete games for which evolution is no more governed by «time» but by «events».



Journals in Game Theory

GAMES AND ECONOMIC BEHAVIOR

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A Foundation of Nash Bargaining Solution from Strategic Uncertainty

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Keywords: *Prudent equilibrium, Nash equilibrium, refinement, strategic uncertainty, better-reply secure.*

The standard Bargaining problem can be summarized as follows: two players have to reach an agreement on the partition of a pie of size 1. John Nash [2] has proposed to axiomatize the desirable properties of such an agreement, in order to obtain one unique solution, called the Nash solution. Then, many researchers have tried to give a strategic (non cooperative) foundation of this concept (see, for example, Walter Trockel [4]), most of them using extensive form games (see Rubinstein [3]). In parallel, many concepts of solutions have been proposed to extend or modify Nash solution concept (e.g., Kalai-Smorodinsky solution concept).

In this paper, we propose a new foundation for the Nash bargaining solution, using *prudent games*, introduced by Philippe Bich [1]. The idea is to introduce a new feature in the bargaining environment, strategic uncertainty, together with some "prudent" behavior of the players with respect to this strategic uncertainty. We prove that in the standard Nash demand game, this prudent behavior induces a unique equilibrium, which corresponds to different kind of cooperative solution concept (Nash's one or Kalai-Smorodinsky), depending on the kind of prudence introduced in the model.

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The Fragmentation of Views in a Democracy

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Keywords: *learning, collective choice, political polarization.*

I consider a model in which two opinion leaders share their signals with the society that faces a collective choice. The model features two environments: one in which both opinion leaders communicate with the whole society and one where each opinion leader communicates only with players who have the same preferences as she does. The first scenario provides higher welfare but the conditions for truthful communication are more restrictive.



Periodicals in Game Theory

GAME THEORY AND APPLICATIONS

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NOVA SCIENCE



Political Bargaining for Regional Distribution of Fiscal Commons in India

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Keywords: *fiscal federalism, redistributive politics, median voter theorem, political economy.*

Distribution of fiscal resources among the sub-national units is one of the most contentious issue in the federal countries. The federal government collects taxes from all sub-national units, generally following a common tax code across the nation. However, federal tax revenues form a common fiscal pool, with incentives for the sub-national units to compete for bringing more of it in their territory. Constitutional design of intergovernmental transfers in India recognized the need to address regional disparities in the fiscal resources of sub-national governments, which arises from underlying regional income inequalities. For this, the constitution created provision for an expert body, namely the Finance Commission, to determine the quantum of resources to be shared between federal and sub-national governments. However, the federal government have used its discretionary powers to channel most of the expenditures relevant for regional welfare, through channels bypassing the Finance Commission. Use of discretionary channels by the federal government has created space for redistributive politics to influence the share of sub-national units in the federal fiscal resources. Recent literature highlights role of political forces in determining share of sub-national units in the federal resources. However, available literature in Indian context has focused on the role of redistributive politics in influencing intergovernmental transfers only. Much larger component of federal expenditures flows outside the channels of intergovernmental fiscal transfers. Present paper addresses this specific gap by using panel dataset of regional allocation of federal fiscal resources relevant for regional welfare. Analysis suggests that while discretionary federal transfers are driven by concerns for regional inequalities, bargaining power of the states through representation in ruling coalition at the federal level also influences disbursement of federal funds. Similarly, coalition partners at the federal government are able to attract more resources to sub-national units ruled by them. The paper also finds some evidence on role of median voter theorem in influencing sub-national distribution of federal fiscal resources.

Certification for Environmental Corporate Social Responsibility and Endogenous Market Structure

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Keywords: *corporate social responsibility, environmental standards, price contracts, quantity contracts.*

The paper analyzes the relationship between the nature of competition and incentive and level of corporate social responsibility standards certified by various agencies (government, non-governmental organizations (NGO)). We consider the choice of price or quantity contracts by firms and its impact on optimal and desired standards chosen by the firms. We show that in Bertrand as well as Cournot competition level of standard set by NGO would be below optimal level and beneficial to both firms and consumers. In a price-quantity game, either only price setting firm would take the certification or both firms will and the Cournot firm would have higher profits than Bertrand firm.



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Nonlinear Pursuit Problem Discribed by System of Order Two

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Keywords: *differential game, nonlinear system, pursuer, evader.*

Differential game $\Gamma(x_0, \dot{x}_0)$ of two players is considered in the space \mathbb{R}^k ($k \geq 2$) and described by next system of differential equations

$$\ddot{x} = f(x, u) + g(x, v), \quad u \in U, \quad v \in V, \quad x(0) = x_0, \quad \dot{x}(0) = \dot{x}_0,$$

where $U = \{u_1, \dots, u_m\} \subset \mathbb{R}^l$ — value set of the pursuer control, $V \subset \mathbb{R}^s$ — compact — value set of the evader control. The pursuer's purpose is a system translation in finite time to any given neighborhood of zero. Pursuer use piecewise open-loop strategies constructed only by using information on state coordinates and velocity in partition points of time interval. Sufficient conditions of capture problem solvability in the piecewise open-loop strategies class is obtained. Also, it is proved that the capture time tends to zero with the approach the initial position to zero. It happens independent of the evader's actions. The solution of this problem based on the positive basis notion [1].

The following theorem is true.

Theorem 1. *Let $f(0, u_1), \dots, f(0, u_m)$ is a positive basis $-g(0, V) \subset \text{Int}(\text{co}\{f(0, u_1), \dots, f(0, u_m)\})$. Then, there exist values $\varepsilon > 0$, $\theta > 0$ and $T > 0$ such that for any initial positions x_0, \dot{x}_0 that inequality $\|x(0)\| + \theta\|\dot{x}(0)\| \leq \varepsilon$ holds, capture occurs in the game $\Gamma(x_0, \dot{x}_0, T)$.*

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Combining Guaranteed Feedback Pursuit Strategies for Alternative Terminations

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Keywords: *singular surfaces, strategies with memory, alternative pursuit.*

Setups of “classical” two-person (P and E) zero-sum pursuit-evasion games include state-control equations for system dynamics, constraints for the state and control variables, descriptions of terminal manifolds and optimality principles that allow to decide which of admissible strategies is the best for P and E (if any). There is no unified formalization where all the components were rigorously described in full detail; see, e.g., [1]. And it doesn’t seem possible that such kind of formalization will appear one day. Moreover, in spite of all developments in the theory of differential games, it’s quite difficult to apply any existing formalization for modeling even basic conflict situations. For example, if E controls a group of separate evaders $\{E_1, \dots, E_n\}$, $n \geq 2$, and P strives to approach them in some succession in minimal time, one can try to build a model as a differential game where P may accomplish the mission by a finite number of variants for approaching $\{E_1, \dots, E_n\}$ as an additional part of a pursuit strategy (there exist $n!$ alternatives). However, there is no known formalization where a strategy may include combined continuous and discrete parts. In this paper we discuss how to combine known guaranteed feedback pursuit strategies for a number of alternatives into non-optimal but featured pursuit strategies with memory; see, e.g., [2-3].

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Less may be Better: On the positive effect of the entry restriction

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Keywords: *Bertrand competition, social welfare, additive preferences, ford effect, Schumpeter hypothesis.*

When firms are sufficiently large, they face demands, which are influenced by the income level, depending in turn on their profits. As a result, firms must anticipate accurately what the total income will be. In addition, firms should be aware that they can manipulate the income level, whence their “true” demands, through their own strategies with the aim of maximizing profits. This feedback effect is known as the Ford effect. In what follows, we shall assume that wage is given. This includes the way proposed by O.Hart, in which the workers fix the nominal wage through their union. This assumption implies that only the profit Ford effect is possible, moreover, firms maximize their profit anyway, thus being price-makers but not wage-makers. The presented paper uses an additive separable utility function of general type. The main focus of this research is on the welfare aspects, e.g., how many oligopolies are needed to foster the Social Welfare? The common wisdom tell us that, *ceteris paribus*, the tougher is competition (or, the more firms enter to industry), the higher will be consumers’ well-being. In monopolistic competitive models the only condition to cease the enter is Zero-Profit condition, which is quite natural for case of myriads small firms seeking the way to earn even very small profit. It would be very expensive for government to control the enter of these myriads. The case of relatively small number of firms (oligopolies) is quite different. Some types of enterprise activity require a governmental license, which may be given or withdrawn by various reasons. Typical examples of such industrial branches are cellular connection, television/radio broadcasting, air traffic, etc. This paper shows that in most cases the limitation of enter may be beneficial for social welfare, in other words, the socially optimum number of competing large firms (oligopolies) is strictly less than number determined by Zero-profit condition under free entry. As a natural policy implication of this result is a governmental limitation of firms’ entrance at proper level. Our results are not so counter-intuitive as they may seem. It is common wisdom that large firms (oligopolies) gathered more market power than perfect\monopolistic competitive firms, which allows them to operate with positive profits, and taking into account the Ford effect increase the market power and the profits even further. Joseph Schumpeter argued that oligopolies spend their profits to R&D activity, to finance various social programs, etc., and the positive externalities of such activity offset social losses from oligopolization. Moreover, to be able to invest into R&D firm needs to be sufficiently large. In our model we don’t use any specific mechanism of the R&D investments, the only assumption is that all oligopolistic profits are eventually redistributed across consumers.

Effectiveness of Conglomerate Alliances in Refining Industries: a Game Theory Approach

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Keywords: *coopetition, Cournot oligopoly, Nash bargaining solution, conglomerate alliances.*

We investigate upon a coopetition agreement between a leader (A), being considered as oil refining giant, a niche player (B), being considered as specialized chemicals producer, and their joint venture (C) within the market of petrochemicals and other chemicals consumers (D). A and B are realizing cooperative strategies in technological competences, while they are acting competitively on the market. Thus, C is considered to be a cooperative manufacturing venture of A and B. Strategies: both players are sharing their manufacturing between their individual core business and joint venture, according to the expected prices' estimates of the products. Then, they are selling final products on the market (D) which is considered to be oligopolistic. The products are non-homogenous.

Assumption 1: such alliance implies both horizontal and vertical integration, so as to the petrochemical segment development is achieved by vertical integration, whereas, more distracted from the core oil refining, other chemicals business is considered to be horizontal conglomerate integration.

Assumption 2: A and B join parts of their capital expenditures within the C in order to proceed with both petrochemicals and other chemicals products. They are shared between the product range, according to the expected price estimate of each of the products. Assumption 3: A and B compete on the market, which is considered to be oligopolistic.

Recently, there has not been yet provided an explicit theoretical solution and an empirical evidence for the conglomerate alliances in oil refining to be effective. In this research we propose a formal solution and a numerical example. We consider effectiveness here to be an external stability in the long-run (Zenkevich, Reusova, 2017). We derive the Nash bargaining solution for several players. The expected gain is considered to be a difference between the total revenue from both individual and cooperative production and fixed/variable costs with the investments in the foundation of a joint venture.

We consider a major oil refining company to conceivably coopete with chemicals producers in order to develop further its petrochemicals business-segment. Thus, we are looking for a theoretical justification on such kind of alliances to be long-term effective.

A Discrete Time LQ Stackelberg Game with Myopic Follower and Linear State Dependent Constraints

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Keywords: *Stackelberg equilibria, linear quadratic dynamic game with constraints myopic follower, fishery extraction, poacher, discontinuous equilibria.*

In this paper, we analyse the simplest linear quadratic Stackelberg dynamic game with myopic Follower and state dependent constraints on strategies.

We consider the feedback information structure of both players, while the Follower also knows the current decision of the Leader, who is the first mover.

This game model e.g. of extraction of a fishery by two players the Owner and a Poacher. The Owner of the lake is far-sighted and he is the Leader and the Poacher is myopic and he is the Follower. The linear constraint are inherent in this framework—each of the player cannot fish out more than currently available in his fishing area.

If the game is one stage only, the informational advantage of the Leader, results in the fact that his strategy and payoff are larger than the Follower's.

If the game has two stages, the far-sightedness of the Leader becomes costly—for certain levels of the initial values of the state variable, his first-stage decision, payoff and value function are substantially lower than Follower's. Besides, the equilibrium strategies are discontinuous for both players and the Leader's optimal strategy is nonunique, although the value function of the Leader is continuous and unique.

Moreover, the value function of the Leader is not monotone in the initial state even in one stage.

By this example, we illustrate difficulties which appear in calculation of Stackelberg equilibria in presence of constraints which may be active at the optimum.

We also compare these myopic-follower-Stackelberg equilibria to the symmetric Nash equilibria in the same game.

Advertising Stackelberg Competition with Application to Bitcoin Market

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Keywords: *differential games, Stackelberg competition, dynamic advertising, contest theory, bitcoin mining pool, bitcoin market shares.*

In this paper, we study the Lanchester model of advertising competition where the attraction rates are taken from the economic theory of contest. Our model has an application in the duopoly of the Bitcoin market where two bitcoin mining pools (leader and follower) compete for the Bitcoin Market share. We solve the game for Stackelberg competition between leader and follower. We compare our results with already known Nash equilibrium case. We obtain that the leader has the advantage to move first and he has substantially larger profit compared to Nash equilibrium. We analysed different scenarios of leader and follower with their graphical interpretation.



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Feedback and Development in Dynamic Tournaments

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Keywords: *tournaments, contests, information revelation, feedback, development.*

We consider a two-period tournament in which agents exert effort in each period and their efforts and productivities are complementary. We study ex-ante optimal feedback policies when the principal privately observes relative performance at the end of the first period. Our model has two key features: (i) at the beginning of the second period, given available information, each agent decides whether to continue working with the same method or to switch to a new one, and (ii) the first-period effort can increase the final outcome if the agent works with the same method in both periods. Without any feedback, whenever agents exert effort in the first period, they have no incentive to try a new method; thus, they keep working with the same method. However, with the performance feedback, they may try a new method when inferring that the productivity with the current method should be too low, which can disincentivize agents from exerting effort in the first period. We show how incorporating this competing effects of feedback affects ex-ante optimal feedback policies that have been identified in the existing studies.



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Integrating Context Effects into Procedural Decision Making

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Keywords: *behavioural Inconsistencies, context effects, limited foresight, procedural decision Making, utility.*

This paper proposes a simple framework to model contextual influences on procedural decision making. In terms of utility, we differentiate between monetary and contextual, psychological payoffs, e.g. deriving from the subjects' normative frame of reference. Monetary payoffs are treated as common knowledge while psychological payoffs are treated as partly unforeseeable. Using this framework, we demonstrate how various common inconsistencies in decision making can be gathered under one abstract pattern. More specifically, we show how several standard models from behavioural economics, such as various forms of social preferences or a present bias, are nested in the proposed framework. Over the last decades experimental economics has convincingly demonstrated that monetary incentives are not all what counts for individual decision making – especially once decisions are related to some form of prominent social context (see Camerer, 2003, for a review). As a consequence, a variety of behavioural models have been proposed to account for effects of fairness and reciprocity (e.g. Rabin, 1993; Charness and Rabin, 2002), other-regarding preferences (e.g. Levine, 1998; Fehr and Schmidt, 1999), regret (Loomes and Sugden, 1982), identity (Akerlof and Kranton, 2000) or a warm glow of giving (Andreoni, 1990). Eventually, all these models add some form of non-monetary component to the utility function and they show how certain effects can be accounted for that way. We propose a simple, more abstract framework to model such contextual influences on procedural decision making. The idea behind this is that a more abstract setting will allow to identify more generally what types of effects may arise and where modelling problems may exist. For our model, we differentiate between monetary and psychological payoffs: Monetary payoffs are treated as common knowledge, while psychological payoffs, which are thought of as integrating various kinds of contextual factors (like e.g. emotions, feelings, social norms and cravings), may be unforeseeable even for the respective decision maker herself. Formally, we use extensive form games as the general basis and introduce an extension which we call context games. In doing so, we keep the common structure of histories but assume that the terminal payoffs as perceived by the players may vary as play moves on in order to account for the context created, for example, by a particular history. We introduce a notion of subgame perfection for context games with perfect foresight. In case of limited foresight regarding psychological effects, however, a different concept is needed as inconsistencies between expected and realised play may arise without being irrational (in the sense of 'crazy'). For these cases we define the notion of procedurally justifiable strategies. These strategies are the result of a procedural decision making process in which the decision maker tries to maximise his payoffs given his information at each stage.

On Control Reconstructions to Management Problems

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Keywords: *dynamic controlled processes, inaccurate state measurements, control reconstruction, concave-convex discrepancy.*

Dynamic controlled processes are widely found in the surrounding life: management in technology, business, economics, medicine, social change, traffic, education, etc. Investigations of the processes lead to the need to study and solve not only direct problems aimed at constructing controls that optimize quality criteria, but also solving inverse problems on the control reconstruction for inaccurate measurements realized motions. We suggest and discuss a new approach to solving of control reconstruction problems in real time. The suggested solution based on necessary optimality conditions to auxiliary calculus of variation problems with concave-convex discrepancy functional. We discuss connections of the new approach with other ones. Results of simulations of the control reconstruction are presenting for a management problem of macroeconomics.



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Competitive Multilateral Selection Problems

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Keywords: *game models, optimal stopping, dynamic programming, secretary problem, sequential search.*

The paper deals with a game version of the cardinal pay-off no-information secretary problem (known also as the Best Choice Problem (cf. Hill(2009)) formulated by Bearden (2006 and reformulated in Szajowski (2009)). There are various experimental studies concerning unilateral sequential choice models. Some of them stimulate the theoretical investigation. It is obvious that modeling competitive selection needs to take into account much more factors than the one person case. The discussion of the issue will be given.

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Social Media Advertising as a Signal of Quality

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Keywords: *quality signaling, opinion leadership, social media marketing.*

Social media advertising has a number of characteristics that distinguish it from other types of advertising, and which may be of key importance in answering the question about its ability to serve as a signal of quality. User participation in content creation and co-creation, as well as emergence and influence of opinion leaders allows this type of advertising to be unpaid for or paid indirectly. On the other hand, the study highlights that the majority of products and services advertised online are experience goods, which means their quality levels cannot be estimated before purchase and consumption. This creates incentives for businesses to use deceptive advertising – thus, understanding the conditions in which social media advertising is a credible signal of quality is crucial to protect consumers. In the game-theoretic model presented in this paper, the monopolist sends an advertising signal to bloggers - opinion leaders in a social network. The latter, in turn, make decisions about placing an advertising message in their blog, taking into account the impact that this action may have on their reputation. The study investigates the question of when advertising can serve as a signal of quality in a separating equilibrium.



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DYNAMIC GAMES AND APPLICATIONS

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Fractional McKean-Vlasov SPDEs for the Fractional Mean Field Games

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Keywords: *fractional McKean-Vlasov SPDE, mean field games, regularity.*

The McKean-Vlasov equations are in the center of modern research in the theory of mean field games (MFGs). On the other hand, the analysis of the stochastic games with memory leads naturally to the necessity to analyze such games via the time-fractional differential equations. The link between these two modelling tools represents the time-fractional McKean-Vlasov equation.

In this work, we are going to present some basic well-posedness and regularity results for this equation, which can be considered as the starting point for the development of the fractional MFGs.



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Consistency and Independence of Zero-Determinant Strategies in Repeated Games

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Keywords: *zero-determinant strategy, repeated games, incomplete-information games.*

Zero-determinant (ZD) strategy, which was discovered by Press and Dyson in 2012, is a strategy that unilaterally assigns a linear relation between average payoffs of all players in repeated games [1]. ZD strategies include equalizer strategy that unilaterally sets average payoff of one player, and extortionate strategy in which the player can get more average payoff than his opponent. While ZD strategies were originally found in two-player two-action games, ZD strategies were extended to multi-player multi-action games [2]. A natural question is "What happens when more than one players employ ZD strategies?". There are two possibilities: linear payoff relations assigned by ZD players have solutions, or they do not have any solutions.

We theoretically prove that linear payoff relations assigned by ZD players always have solutions. Furthermore, under a general condition, we prove that linear payoff relations assigned by players with ZD strategies are independent, which means that there does not exist a ZD player whose ZD strategy adds no linear constraints other than those already imposed by other ZD players. These results hold in general multi-player multi-action repeated incomplete-information games [3].

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An Approach Problem with an Unknown Parameter and Inaccurate Motion Measurement

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The paper is devoted to the study of the approach problem for a nonlinear control system with a compact target set in a finite-dimensional phase space of the system. A special feature of the problem considered in the present paper is the presence of an unknown constant parameter in the system. It can be treated as a game-theoretic approach problem, where the first player aims to bring the system closer to the target set, while the second player, who can choose the value of the parameter, seeks to keep the first player from achieving this aim. Taking this view of the problem, we can extend the class of program controls of the first player to the class of positional controls and can embed the class of second player's strategies in a wider class, for example, the class of positional strategies of the second player, and then we can treat the approach problem as a positional game-theoretic approach problem. Solving the problem in the framework of positional formalization gives back the set of positional absorption as the resolvability set of initial positions, and for all initial positions in this set the extremal positional strategy can be taken for a solution strategy. This extremal strategy would also guarantee a solution of our original approach problem, in which the value of the parameter remains constant up until the terminal time. Using this approach, we would not need to turn to the procedure for identifying the parameter as one of the main steps in solving the problem. However, we would not obtain a full solution of the original approach problem with this method, because in general the set of positional absorption is narrower than the solvability set of the original problem. As we want to obtain a full solution in our paper, we will not use a reduction of the approach problem: we will construct an (approximate) solution of the problem based on its special features. On our way we have to recover the value of the parameter on some small initial interval of time [1].

The study was supported by the Russian Science Foundation (project no. 19-11-00105).

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Price Advertising Game in Duopoly with Network Externalities in Consumption

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Keywords: *price advertising, externality, search, network effect, oligopoly.*

The paper presents a duopoly model with two firms producing a good with network externalities in consumption. Consumers cannot easily learn prices and thus they are not able to correctly form their expectations about the potential clientele size. Consumers may either search to observe the prices or learn them from firms if there is price advertising. At first the duopolists decide whether to advertise and then compete in prices. Advertising firm commits to its price with public announcement. Consumers possess random utility which actual value depends on the horizontal match of the goods. Therefore consumers must visit the stores to inspect the goods, realize their horizontal matches, and learn the prices if there was no price announcements. Each searching consumer incurs one-time search cost. There are three possible outcomes: both firms remain silent; both firms advertise their prices; only one firm advertises. Each outcome implies different pricing strategies and market sharing. Since, in general, the model predicts multiple equilibria, after refinements a few equilibria are selected. In these equilibria, a decision to advertise prices crucially depends on the type of the consumption externality and the value of the search cost. Particularly, at least one firm must advertise for a negative externality, as it lowers the minimum threshold level of search costs and increases the profits of the advertising firm compared to the case with no disclosure. Further, both negative and weakly positive externalities lead to equilibria with interior solutions where both firms have positive market shares and neither firm is able to capture a whole market. When consumption externality exceeds the degree of product differentiation, the demand function of each firm becomes upward-sloping; a strong positive network externality dominates the price effects and therefore a higher price is necessarily associated with a greater market share. Finally, the only equilibria with a strong positive consumption externality are those in which only one firm supplies the entire market.

Super-Differentials for Some Classes of Fuzzy TU Cooperative Games

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Keywords: *totally positive game, fuzzy TU cooperative game, core, super-differential.*

In the paper, we consider conditions providing coincidence of the core and super-differential of fuzzy cooperative game v with side payments [1]. It turned out that one of the most simple sufficient conditions consists of local homogeneity of v . Moreover, by applying so-called S^* -representation of fuzzy game [2] we show that for any v with $C(v) \neq \emptyset$ there exists some game u such that the core of v coincides with the super-differential of u . By applying sub-differential calculus [1] we describe a structure of the core for both classic fuzzy extensions of the ordinary cooperative game (e.g., Aubin [1] and Owen [3] extensions) and for some new extensions, like Harsanyi extension and generalized Airport game.

To give more details, remind first that vector $x \in \mathbf{R}^N$ is said to be a *super-gradient* of a fuzzy TU cooperative game v at the center of gravity $e_N^* = (1/n, \dots, 1/n)$ of the simplex $\sigma_F^* = \{\tau \in \mathbf{R}_+^N \mid \sum_N \tau_i = 1\}$ if for any fuzzy coalition τ it holds $x \cdot (\tau - e_N^*) \geq v(\tau) - v(e_N^*)$.

Definition 1: Super-differential $\hat{\partial}v(e_N^*)$ of a fuzzy game v at the coalition e_N^* is the set of all super-gradients of v at the point e_N^* (below, as usual, $\sigma_F = [0, 1]^N \setminus 0$)

$$\hat{\partial}v(e_N^*) := \{x \in \mathbf{R}^N \mid x \cdot (\tau - e_N^*) \geq v(\tau) - v(e_N^*), \quad \tau \in \sigma_F\}.$$

Definition 2: Cooperative game v is said to be locally homogeneous at e_N^* if there exists $\delta > 0$ such that $v(te_N^*) = tv(e_N^*)$ for any $t \in [1 - \delta, 1 + \delta]$.

It can easily be checked that for any locally homogeneous at e_N^* game v with $C(v) \neq \emptyset$ we get $C(v) = \hat{\partial}v(e_N^*)$. To obtain more valuable result consider an arbitrary fuzzy game v . Take its S^* -representation v^* [2] and by \hat{v} denote *homogeneous extension* of v^* defined by the formula $\hat{v}(t\tau^*) := tv^*(\tau^*)$, $\tau^* \in \sigma_F^*$, $t\tau^* \in \sigma_F$.

Theorem 1: For any V -balanced fuzzy TU cooperative game v it holds

$$C(v) = \hat{\partial}v(e_N^*).$$

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Super-Differentials for Some Classes of Fuzzy TU Cooperative Games

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Keywords: *repeated game, variable discounting, subgame perfect equilibrium, Pareto optimal outcome.*

We consider a class of repeated games with side payments, complete information on prehistory and variable discounting, in particular, with sliding planning horizons. This class of supergames is of general interest because in many real repeated conflict situations the relative weights of future payoffs in individual utility functions change with the time. In particular, in many countries decision making persons regularly are changed or stay the same depending on the election results. Then indicators of the socio-economic situation related to the time before the closest election are often much more important for persons in power than subsequent indicators. Moreover, many people, when planning their future activities, consider the consequences only till some finite planning horizons. As the time passes, these horizons also move. So the repeated games under consideration reflect incomplete rationality of participants, typical for many real conflicts. Below we aim to examine how discounting coefficients (in particular, planning horizons) influence the set of Pareto-optimal subgame perfect equilibria. The initial game includes two stages. The first stage is a game that generalizes the known model “The Tragedy of the Commons” (Hardin, 1968). At the second stage the players redistribute the payoffs by means of side payments. Our purpose is to find conditions for existence of a subgame perfect equilibrium (SPE) realizing in the repeated game some Pareto-optimal outcome, that is, an outcome that maximizes the total payoff in the one-shot game. Note that in the one-shot game there is a “bad” Nash equilibrium in dominant strategies corresponding to a high pollution level. We examine two types of SPE realizing some Pareto-optimal outcome: 1) after any deviation, all players start playing dominant strategies; 2) if one player deviates, the rest continue cooperation maximizing their total payoff under the dominant strategy of the disturber; after the second deviation everybody plays his dominant strategy. For each type, we determine the set of Pareto-optimal SPE outcomes. Variant b seems to induce more rational behavior of the players since, after the first deviation, the rest players continue cooperation and maximize the total welfare. However, the conditions for existence of the SPE are more strict for this case and do not hold in general even under long planning horizons (in contrast to variant a). The problem is of interest in context of the study of international agreements on limitation of environmental pollution (see Petrosjan, Zaccour, 2003, Breton et al, 2018). Existence of the SPE means the possibility for a stable and efficient agreement of such sort. This concept takes into account a possibility of unexpected breaking the agreement by some country and assumes that only endogenous economic mechanisms in frame of the agreement prevent such breaking. Side payments reflect possibilities for redistribution of the welfare among the players.

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Information Disclosure in Elections with Sequential Costly Participation

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Keywords: *voting, participation, information disclosure.*

Electoral legislation varies across countries and within countries over time as well as across different types of elections with respect to its stance to making intermediate election results such as turnout and candidates' vote shares public knowledge during election days. In some countries, electoral authorities every few hours announce cumulative turnout but do not allow for disclosure exit polls results until all polling station are closed. Some countries allow for disclosure of any relevant information including results of exit polls at any moment, even when polling stations are still open. In others, any information disclosure until the end of election day is illegal.

In this paper, using a pivotal costly voting model of elections where voters have privately observed preferences over two candidates and act sequentially, I study how different information regimes (no information disclosure, turnout only disclosure, votes disclosure) affect voters' decisions to cast votes and what impact they have on voter and candidate welfare, depending on candidates' ex-ante support. I find that first, full (votes) disclosure dominates the other two regimes in terms of voter welfare. Second, partial (turnout) and no disclosure quickly converge with larger number of voters in terms of both voter and candidate welfare. Third, ex-ante majority candidate has the highest chance to win under full disclosure, minority candidate with relatively high support - under partial disclosure, and minority candidate with low support - under no disclosure.



Periodicals in Game Theory

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Multiple-Population Discrete-Time Mean Field Games

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Keywords: *mean-field game, discrete time, discounted reward, total reward, multiple populations, stationary mean-field equilibrium.*

In the paper we present a model of a discrete-time mean-field game with several populations of players. Games of this type have been studied in literature only in continuous-time setting. We consider two payoff criteria: discounted reward and total reward. For both, we present the results about the existence of a stationary mean-field equilibrium in this type of games. We also provide some results linking equilibria in the mean-field models with approximate equilibria in finite-player games defined with the same primitives.



Journals in Game Theory

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SPRINGER



Consistency of the Equal Split off Set

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Keywords: *cooperative games with transferable utilities, the Dutta–Ray egalitarian solution, the equal split off set, consistency properties of cooperative game solutions.*

A wellknown egalitarian solution for convex transferable utility games was defined in the seminal paper of Dutta and Ray (1989). This solution can be considered as a compromise between egalitarianism and selfish behavior, where egalitarianism is formalized by Lorenz domination and selfish behavior is modeled by core-like participation constraints. As a consequence, the Dutta and Ray (1989) assigns to any convex game the unique Lorenz dominating core allocation. Hougaard et al. (2001), Arin and Inarra (2001), and Arin et al. (2008) extended this approach to balanced games and analyzed Lorenz undominated core allocations. Branzei et al. (2006) introduced the equal split-off set (ESOS) as an extension of the DR solution to all transferable utility games by the extension of the algorithm of Dutta–Ray to arbitrary games. This algorithm recursively assigns the maximal average worth to the members of one of the corresponding coalitions, and subsequently considers a reduced game for the other players. It turns out that the ESOS is nonempty and efficient solution, but, in general, it is set-valued. Nevertheless it keeps some egalitarian properties even for unbalanced games. Sanchez-Soriano et al. (2010) showed that all equal split-off set allocations Lorenz dominate all (other) core allocations. Axiomatic characterizations of the ESOS are connected with the consistency properties of the TU game solutions and with the solutions of two-person games. For the super-additive two-person games the DR solution coincides with the solution of the constrained egalitarianism (CE). We extend the CE solution to sub-additive two-person games with the help of the Dutta–Ray algorithm that can be applied to sub-additive games as well. Consistency properties of the DR solution for convex games (Davis-Maschler, complement consistencies and some other ones) are not satisfied for arbitrary TU games. However, the ESOS satisfies some restrictions of these properties that turn out to be sufficient for characterizations. This presentation axiomatically studies the ESOS on multiple classes of non-convex games. We obtain several new characterizations by exploring the relation of the equal split-off set with various consistency notions in different forms.

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A Game Theory Model Applied to After-Sale Services

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Keywords: *after-sale services, customer's maintenance effort, expected payoff, Stackelberg game.*

The after-services are a set of activities dedicate to provide support to the buyer when he buys a product or a service, meaning an insurance of the item purchased. For this kind of environment, there are at least two players; the consumer and the original manufacturer equipment (OEM), where both have different points of view about this extra service (may be conflicting in some cases). Thus, this paper aims to show a game theory model applied to after-sale services when there are two players; a consumer, the owner of the piece of equipment and receiver of after-sale services and the OEM who is responsible for producing, selling, and providing the after-sale services. Moreover, there is an important variable that affects players' payoff, the customer's maintenance effort. The model is solved by the Stackelberg solution, through an optimization system divided into two steps whose the leader is the OEM who defines the prices of device and insurance level. Finally, an example illustrates how the model works, emphasizing the key elements and the decision variables of the players.



Journals in Game Theory

GAMES AND ECONOMIC BEHAVIOR

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Competitive Pricing for Cloud Information Resources

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Keywords: *pricing, cloud resources, two-stage non-cooperative game, Nash equilibrium.*

This article explores a pricing model for cloud resources, based on the use of two different payment schemes - reservation and pay-as-you-go, each of which is controlled by its administrator. Interests of administrators and customers are both in the scope. Scheme administrators select prices in order to maximize their own revenues. At the same time, the task of clients is to choose the payment scheme with the least possible expected costs. The process of prices determination has a form of a two-stage game. First stage is a static non-cooperative game between the administrators for the opportunity to sell resources, where each administrator assigns the price in order to maximize his expected revenue. We use M/M/ ∞ and M/M/1 queues to take into account the correlation between service times and the flow rates of requests for the reservation and pay-as-you-go schemes, respectively. As a result, we derive sufficient conditions for the existence of a Nash equilibrium. In the second stage, competition among customers who wish to purchase cloud resources is studied. When the prices are set, we analyze clients choices of schemes. Making a choice they seek to minimize their expected costs, which consist of financial component and waiting costs. Wardrop principle is used in order to describe user behaviour and optimality conditions. At the end of the work, it is shown that implementation of the additional reservation scheme has positive effect for the provider and the clients compared to a single pay-as-you-go-scheme. A numerical simulation of pricing is carried out for various values of parameters in order to determine the degree of influence of various factors on the equilibrium values of prices and utilities. The remainder of the article is organized as follows. Firstly, a review of the subject area is made. Secondly, an overview of the scientific literature on cloud resources pricing is provided. Then pay-as-you-go and reservation schemes model is described. Further goes examination of price competition among scheme administrators, as well as resource procurement competition between customers when choosing a payment scheme; a comparison of the case of one and two schemes in equilibrium is carried out, an analysis of the results of numerical simulation is given. Conclusion and perspective ways for future research are given in the end.

Key Players in the Distribution Network Design under Dynamic and Complex Situations

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Keywords: *key players, key leaders, game theory, Nash equilibrium, distribution network design.*

In organization studies and strategic management, particularly in a supply network context, the investigation of roles, the importance of players (organizations) and their relationships have attracted the interest of many scholars with respect to their influences on such issues as a performance enhancement, innovation adoption, etc. The result of these studies can be applicable for supply network managers to take into account in their network design and modification tasks. It is a critical practice that affects the performance of both players and the network. There is a dearth of research and empirical studies, which have paid attention to the concept of key players and key leaders as an organization at the network level of analysis, especially in the logistics network context. It is due to the complexity of both disciplines, which should be considered in a dynamic situation. Also, the term of power which is most popular as an interrelated feature with leadership to describe how an industry dominates over other agents in the network can be defined differently based on the definition of performance and network objective in each problem. Identifying the key players and key leaders in a network are one of the primary uses of network analysis. There are some metrics to show how the networks were organized such as network density, centralization, and complexity. The concept of centrality is a fundamental criterion for measuring the importance of players based on the theory of power from a network perspective. In recent studies related to key players and key leaders in social networks (Zhou and Chen, 2015; Zhou and Chen, 2016; Ballester et al., 2006) the weighted Katz–Bonacich Centrality (Bonacich, 1987; Katz and Shapiro, 1985; Bonacich and Lloyd, 2001; Bloch et al., 2017) have been the main criteria, which also in this research will be considered for measuring key players and key leaders with the implementation of Stackelberg leadership model (Von Stackelberg, 1952) and the Nash equilibrium concept (Nash, 1951). In this paper, a distribution network, which is intermediary level in supply networks and facilitates the physical flows of merchandises between suppliers, manufacturers, distribution centers, and retailers, is under investigation based on the investment of each player (DC) in vehicles as a strategic decision to increase their profits in a non-cooperative interaction. The importance of this decision to decrease transit time, inventory costs, security and availability enhancement, and environmental impact can be studied in the literature (e.g., Coyle et al., 2016; Gubbins, 2003). Therefore, the total profit is the measurement of the payoff function. A well-known online store in Iran has decided to extend its market and scope of work among all 31 provinces throughout the country.

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