Ex ante effects of ex post managerial ownership*

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ABSTRACT
This paper highlights the trade-off between the need to restructure a company and the need to provide managers with appropriate incentives to run it after the restructuring. In order to provide incentives, it is optimal to let managers acquire equity in the firm. However, the expectations to be able to buy shares in the future may create ex-ante incentives to delay restructuring. This effect is particularly important for events where managers can acquire a substantial number of shares, such as privatizations or MBOs. In equilibrium, the shares are not underpriced, but the delay in restructuring or the loss of good investment opportunities which took place in the period before reduces the value of the company. We report empirical evidence on MBOs and privatizations consistent with the model in this paper.

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1. Introduction

The corporate finance literature has often stressed the importance of managerial ownership. Jensen and Meckling (1976) show that separation between ownership and control usually leads to agency costs, since managers do not necessarily have the maximization of the value of the firm as their primary objective. These agency costs imply that, if possible, it is optimal to give managers some equity (or equity-like securities, such as stock options) in the company, since it will align their objectives with those of the shareholders. There is a large literature on managerial compensation with equity and stock options that follows from this principle.\(^1\) Moreover, several papers present empirical evidence that corporate performance increases with the level of share ownership by corporate insiders (see, for example, Morck, Shleifer and Vishny (1988), McConnell and Servaes (1990) and McConnell, Servaes and Lins (2005)).\(^2\)

When we move to a multi-period framework, expectations of future actions can affect the analysis, and we need to take into account the \textit{ex ante} effects. For example, Jensen and Meckling (1976) point out that expectations of future sales of equity will have an \textit{ex ante} effect on the decisions taken today by the managers. Bar-Gill and Bebchuk (2004a) look at the \textit{ex ante} effects when managers know they will be able to sell shares if they receive negative information about the firm’s prospects. In this paper, we look at the \textit{ex ante} effects on managers’ decisions, when they expect to be able to buy additional shares in the future. We take into account the fact that additional ownership increases managerial incentives, so that the \textit{ex post} value of the firm increases with the manager’s equity fraction. However, from the \textit{ex ante} point of view, the prospective to acquire shares in the company provides perverse incentives to managers. Managers may not undertake good investment opportunities or delay restructuring, in order to be able to buy cheap shares and acquire a larger stake in the company.

\(^1\)See Murphy (1999) for an overview of the literature.
\(^2\)To be precise, the relation is non-monotonic, as corporate performance starts decreasing beyond a certain level of insiders’ share ownership. This can be explained by corporate insiders extracting private benefits from control and is beyond the scope of this paper.
In our paper, the negative ex ante effect takes place not because managers will buy shares at a price which does not reflect the true value of the shares (as in Jensen and Meckling (1976) or Bar-Gill and Bebchuck (2004a)). On the contrary, prices will reflect the true value of the firm’s assets: when managers buy the shares, they buy them at the fair price (taking into account all future opportunities) and therefore do not make any extra profits. Despite that, ex ante they have an incentive to distort their actions. In this sense, the paper has a flavor similar to Stein (1989), where managers, trying to fool the market, forsake good investments in order to boost short term earnings, but in equilibrium the market perfectly adjusts its expectations and is never fooled. Also in Goldman and Slezak (2006) managers has an incentive to manipulate information, despite the market adjusting for the bias. In Holmstrom (1999) managers exert extra effort in order to convince potential future employers that they are of a better ability, but in equilibrium their true ability is known.

We identify three different effects, in increasing order of importance, that distort managerial incentives. We start from the usual Jensen-Meckling (1976) effect: if managers do not own the entire firm, they do not internalize all the benefits arising from their effort and therefore a suboptimal amount of investment is undertaken. We then show that such effect is aggravated when managers can buy shares. In fact, the increase in the firm value due to managers’ effort is fully reflected in the share price other investors may be willing to pay. As a result, managers will pay a higher price for the shares and will never be compensated for their effort and will want even less to undertake new investment. Finally, we introduce asymmetry of information at the ex ante stage and show a third effect: managers have all the incentives to bias the information of other potential buyers (or sellers) and therefore their investment decisions in the first period may be directed to garble information. In equilibrium, they do not manage to fool the investors and by the time they can buy the shares, the share price is the correct one, but the damage (the lack of restructuring or the loss of good investment opportunities) has already taken place. The total amount of underinvestment in equilibrium is determined by the sum of these three effects.
This trade-off between \textit{ex ante} and \textit{ex post} incentives may arise even on a daily basis, as managers can acquire more shares in the market. However, it is particularly important and can lead to severe inefficiencies in situations where managers can increase their ownership stake substantially. Therefore, this paper focuses on two types of transactions: management buy-outs (MBOs) and privatizations. In both cases, managers have the opportunity to increase substantially their ownership in the firm. Moreover, in both cases a substantial restructuring is needed and it is important that the right incentives are in place in order to induce the managers to undertake the restructuring.

Usually, an increase in managerial ownership is seen as a positive implication of an MBO or privatization, and quoted as one of the main causes of the observed increase in the productivity of a company following an MBO or a privatization. However, the \textit{ex ante} perspective may be less favorable. In fact, these are usually situations where the company could benefit substantially from major restructuring and we show that the expectation of being able to buy additional shares may delay crucial restructuring and affect the long term value of the firm. Moreover, if good investment opportunities arise after the privatization has been announced, or after the managers have decided to undertake an MBO, managers may not want to undertake them.

Given that managerial ownership remains a good thing in the long run, this paper highlights the need to pay more attention to the period before these events (MBO or privatization) take place. For example, in the case of privatizations, this paper implies that firms could do worse when privatization is announced than when privatization is not announced and not expected. In fact, the period between the announcement and the actual privatization can be very long and during this period managers anticipating privatization may intentionally slow down and forgo precious strategic opportunities to restructure, since this allows them to benefit more from the privatization. Royal Mail, the public mail service in U.K., has recently announced the intention to conduct

\footnote{For example, Gupta (2005) looks at the case of partial privatizations in India, where control was still retained by the government, and documents an increase in productivity that cannot be attributed to control issues, but rather to better managerial incentives.}
a worker buy-out as a partial privatization.\textsuperscript{4} The date when this will happen (and whether indeed it will happen) has not been announced yet, but all the \textit{ex ante} perverse effects can already take place.

In the case of MBOs, the delay between announcement and implementation is endogenous, since the managers themselves choose when to announce the MBO. However, this feature may only make matters worse, as managers may plan an MBO without announcing it and thus start underperforming even before the announcement. A firm should require managers to inform the board immediately of their intention of undertaking an MBO. Several recent articles in the Wall Street Journal point out how managers did not inform the board for a long time that they were in talks about a possible MBO.\textsuperscript{5} This paper shows how in this period the managers have an all different set of incentives, and the board should be made aware of it as soon as possible.

Another implication of this paper is that this “transition period” (between when a privatization or an MBO is decided and when they take place) may require a different set of compensation rules and incentives, since the managers incentives are different from the ones prevalent in other periods. For example, the government may want to return a fixed proportion of the privatization revenues to the managers, or to reward the managers according to the firm’s share value at privatization. Similarly, in the case of an MBO, managers’ remuneration contingent on the final result of the MBO may reduce the \textit{ex ante} incentives to underinvest. In this paper we do not derive the optimal mechanism to solve the problem highlighted, since sometimes a very simple rule can solve the problem. Rather, we want to highlight the problem: the reason these simple rules are often not adopted is that nobody pays attention to the \textit{ex ante}


\textsuperscript{5}See, among others, “The Game: Fine Line of Selling, Selling Out, the Firm—Bold steps by CEOs get board firing back”, by Dennis Berman, 30 January 2007; “Clear Channel Buyout Talks Fuel Concern of Management Conflicts”, by Sarah McBride and Dennis Berman, 14 November 2006; and “Backstory of Kinder LBO Underscores Web of Ethical Issues Such Deals Face”, by Dennis Berman and Henry Sender, 29 September 2006. It is also mentioned that “a group of top managers was forced out of their jobs, after helping to launch a management-buyout plan for a utility called Alinta Ltd. The conflicts of interest were just too great, the company’s board said”.

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In Sections 5 and 6 we report empirical evidence on privatizations and MBOs that is consistent with our story. In particular, for privatizations, Lopez-de-Silanes (1997) shows that firms being privatized sell at a higher price the shorter is the period between the decision to privatize and the actual privatization. He interprets this result by saying that “managerial distraction and forgone investment opportunities are possible explanations for the loss in value of these firms.” This is exactly the effect we obtain in this paper, but we point out the main cause of this behavior: managers may be already thinking about their future opportunity to buy shares. Moreover, he finds that when the management is substituted at the onset of the privatization process by a professional “privatizer” the price of the shares at the privatization is higher (and this reflects a true better value of the company). Thus, by eliminating the managers the government eliminates all the perverse \textit{ex ante} incentives. There is also abundant anecdotal evidence from transition economies that managers delayed restructuring before a privatization and attempted to make the company to look as bad as possible.

For MBOs, Fidrmuc, Roosenboom and van Dijk (2007) compare management vs. non-management led buy-outs and find that MBOs are characterized by a larger fraction of revenues used for operational expenses in the period before the MBO. This suggests that managers have less incentives to undertake capital investments, as predicted by our model. Moreover, recent literature in accounting, showing that managers tend to manipulate earnings downwards before an MBO, is also consistent with the main idea of the paper.

Our model also has several empirical implications. The first implication is that the substantial productivity increases highlighted by the empirical literature following

\footnote{Although simple measures can sometimes attenuate the problem, we want to stress the solution is not always simple. For example, one may think that to solve the problem in the case of privatization it may be enough to prohibit the managers to buy shares in the privatization or may choose to sell share to them at a fixed price pre-set in advance. However, these measures may not be enough, if managers can buy shares later on the market.}
an MBO (see Kaplan (1989)) or a privatization (see Megginson and Netter (2001) and Megginson, Nash and van Randenborgh (1994)) may be overstated. In fact, at least part of the increase may be due to the artificial downward distortion of the firm’s productivity before the event. Moreover, there should be a difference in the productivity change of management vs. non management led buy-outs. MBOs where there was an institutional investor already in place will have a smaller increase in productivity (since there was less possibility to run down the firm beforehand). We should also observe, before an MBO announcement, an abnormal growth in wages, less layoffs, and an increase of expensed vs. capital investments. Finally, our model implies that in periods where there is a boom of MBOs, there could be an economy-wide effect of managers’ demotivation (since the probability they may be able to conduct an MBO increases).

Our analysis can be applied also to managers compensation in general. For example, CEOs are typically awarded stock option with an exercise price equal to the stock price on the award date. Thus, the managers have an incentive to underinvest in the company, in order to keep the price of the shares low until when they are granted the options. The incentive to underinvest is thus similar, in spirit, to the finding that CEO manage the disclosure of information around the timing of stock option awards (see Aboody and Kasznik, 2000).

Some other papers have highlighted, in different contexts, the trade-off between ex ante and ex post incentives of managers. Acharya, John and Sundaram (2000) highlight a trade-off between ex ante and ex post incentives in the context of resetting executive stock options. Narayanan (1996) shows that all-cash and all-stock contracts induce the managers to, respectively, underinvest or overinvest in the long run. If he receives only cash he may want to boost current cash flow to convince investors he is more able than he really is. If instead he receives only stock that cannot be traded immediately he has an incentive to overinvest, reduce investors’ perception of his ability and being paid with undervalued stock (which is equivalent to being overpaid). Attari and Banerjee (2005) look at a situation where managers have a
control stake in the company and have to decide whether to undertake a positive NPV project. If undertaking the NPV project requires issuing new equity, managers may forego to undertake the project to avoid the dilution in their equity stake and thus a loss of control. Bar-Gill and Bebchuk (2004b) show that the opportunity to sell shares \textit{ex post} creates \textit{ex ante} incentives to set up misreporting opportunities.

The next section presents the model, Section 3 gives the main analytical results. In Section 4 we extend the model and discuss more general issues. Section 5 and 6 discuss the existing empirical evidence for, respectively, MBOs and privatizations and relate it to our theoretical results. Section 7 concludes.

2. The Model

The timing is the following. Period 1 is the period before managers have the opportunity to buy shares (as in a privatization or an MBO). At the beginning of period 2 the MBO (or the privatization) is implemented and managers can buy equity.\footnote{Note that period 2 does not need to be fixed in advance, it could just be an event that is expected to happen sometimes in the future with a certain probability. As the interval between period 1 and period 2 (i.e. the length of period 1) becomes longer the problem stressed in this paper becomes worse.} From period 3 onwards, the firm operates as a normal private firm, with profits $\pi_t$ each period.\footnote{Alternatively, the firm life could terminate at time 3. We will show in footnote 14 that this would not change much in the analysis.} In the main part of the paper, we assume that the manager of the firm does not change following the MBO or the privatization. In Section 4 we consider the case in which the management changes or the possibility that other investors may try to make a takeover offer for the firm in period 1.
Two kinds of economic players are involved in the model. The managers of the firm and the outside investors, who are willing to buy or sell shares of the firm given an appropriate share price. Without losing generality, we assume that all economic agents have the same discount rate $\delta$.

In the first period, a good investment opportunity arises and managers decide whether to take it and make an investment of size $K$. Alternatively, the managers can restructure the firm, where restructuring may take many forms: it may be accumulation of new capital or it may be accumulation of more intangible assets, like knowledge of the market or investment in human capital. Also in this case we represent the restructuring process with the choice of capital $K$.

Therefore, $K$ can be interpreted as a unique investment opportunity which may arise in period 1 or a restructuring the firm may need. The main difference is that if $K$ is a unique investment opportunity then once the managers let it pass it will not be available any more. If instead $K$ represents restructuring, then one may argue that the restructuring may be undertaken after the managers have bought shares (although possibly at a higher cost). The model could easily be changed to allow for managers to accumulate more capital later: indeed, such modification would only strengthen our result (i.e. the manager would have even more incentives to delay restructuring if they can always restructure after period 2). In Section 4.6 we briefly illustrate how the model would change if managers can undertake the restructuring.

\footnote{The model can easily be extended by assuming that the investment opportunity $K$ arises only with some probability. No result would change.}
$K$ after privatization at a higher cost, and argue that our main results would still be true.

The investment $K$ is observable, but not verifiable.\textsuperscript{10} This is a standard assumption in the literature, since it is difficult to describe in a contract what are the good investment opportunities that should be undertaken or what is the optimal amount of restructuring. However, note that our conclusions do not rely on the assumption that $K$ is not contractible. Our argument is that at present in real life managers are not compensated on the basis of $K$ and as a consequence the problems highlighted in this paper (underinvestment and delay of restructuring) arise. One implication of our paper, therefore, will be that if $K$ is contractible companies should put in place contracts contingent on $K$ in the period before an MBO or a privatization (although, as argued in footnote 11, this will not eliminate completely the underinvestment).

The payoff function of the managers in period 1 is

$$U_1(m, K) = m - cK^2$$

where $m$ is the monetary payment to the managers and $K$ is the effort necessary to undertake the investment $K$ (in other words, for simplicity $e(K) = K$). For the main part of the paper, we assume managers receive a fixed wage in period 1, we relax this assumption in Section 4.\textsuperscript{11}

\textsuperscript{10}Of course, in reality some form of human capital investment may not be observed. However, it seems to us realistic to assume that outsiders can observe, although imperfectly, when restructuring or some investment is undertaken, especially if it is a major investment.

\textsuperscript{11}We show in Section 4 that assuming that the managers’ compensation is contingent on the performance of the firm does not change the results, since the amount of restructuring in period 1 reduces period 1 profits. Alternatively, as mentioned above, one could assume that managers’ compensation is contingent on the amount of restructuring undertaken $K$, but it is not clear such a contract would be verifiable. In a previous version, we allowed payoffs of the managers in the first period to be contingent on the first period investment. This introduces an additional source of trade-off for the managers incentives and makes the analysis more complex, but the main result (i.e., managers underinvest) remains true.
From period 3 onwards, the firm will earn in each period $t$ profits

$$\pi_t = \pi = K\theta + e$$  \hspace{1cm} (2)$$

Thus, the value of the firm depends on three factors: the fundamental value of the firm $\theta$, which is given and intrinsic to that firm, the capital $K$ accumulated in period 1 and the effort $e$ exerted by the manager in each period $t$ (starting from period 3). We are implicitly assuming that the amount of capital accumulated in period 1 has a very crucial role, since it affects the profits of the firm from then onwards.

The parameter $\theta$ captures both the tangible and intangible value of the physical assets of the firm: the “potential” of the firm. For simplicity, we assume that $\theta$ is fixed and does not depreciate over time.

The manager knows the value of $\theta$ while the outside investors observe neither effort $e$ nor $\theta$. However, outside investors have an a priori distribution of $\theta$ and they can update their belief about $\theta$ after observing a restructuring $K$. The prior is that $\theta$ is distributed according to the function $F(\cdot)$, with support $[\underline{\theta}, \overline{\theta}]$: $\theta \sim F(\theta) \in [\underline{\theta}, \overline{\theta}]$.

In period 2, the managers acquire a fraction $\alpha$ of the shares, where $\alpha$ will be determined endogenously. For simplicity, we assume that managers can buy a maximum fraction of shares $\bar{\alpha}$. In each period from date 3 onwards, the payoff to the managers is

$$U_t = \alpha \pi_t - ce_t^2.$$  \hspace{1cm} (3)$$

Of course, the managers compensation can still include a fixed wage $m$: since it will not affect any of the derivations, for simplicity we omit it.

\footnote{In a previous version we assumed no such limit, but simply that managers have a wealth (or a credit limit) $w_0$, so that the maximum number of shares they can buy is determined endogenously by their budget constraint. While such assumption is undoubtedly more realistic, it requires much more cumbersome computations. We therefore chose to use this simplifying assumption that allows us to convey the same idea in a simpler way. For generality, we will mention some of the results of the alternative model in later footnotes (quoting the “model with a wealth constraint”).}
3. The Restructuring Choice

The managers can anticipate future payoffs as a function of the number of shares they will own \( \alpha \) and their expectation influences their decision on how many shares to buy. In period 1, they choose how much restructuring \( K \) to undertake, taking into account that this will affect what will happen in later periods. In the following analysis, we solve the model backwards.

3.1. The Ex-post Operation of the Firm

From Period 3 onwards, the manager owns \( \alpha \) shares. If \( e \) is the effort level of the manager, then in each period he maximizes the expected payoff:

\[
\max_e U(e) = \alpha \pi - ce^2 = \alpha (e + K \theta) - ce^2.
\]

the effort level is

\[
e^*(\alpha) = \frac{\alpha}{2c}.
\]

This is the effect underlined by Jensen and Meckling (1976): a higher fraction of equity induces managers to exert more effort, which increases the value of the firm. A higher \( \alpha \) induces higher effort level and \( e \) is linear in \( \alpha \).

3.2. The purchase of shares by the manager.

At time 2, both outside investors and managers can buy shares. Without loss of generality, we normalize the number of shares to 1.

We allow for the possibility that managers are given a (percentage) discount \( d \) on the price of the shares.\(^{13}\). We also allow for the possibility that managers receive a fraction \( \alpha_0 \) of the shares for free (in addition to those the managers will buy). Note

\(^{13}\)For example, a discount was offered to managers (and workers) in the Russian privatization, see Boycko, Shleifer and Vishny (1993).
that $\alpha_0$ can also be interpreted (especially for the case of MBOs) as the shares managers already own at the beginning of period 2. This highlights some similarities that the present paper has with the literature on toeholds (see Shleifer and Vishny, 1986). We consider both possibilities and study the different effects that these measures have.

The manager can buy shares in the company in many different ways, depending on the specific event we are analyzing. In Section 4 we consider various possible mechanisms. In this section, we assume that the price managers pay for the shares is the reservation price of outside investors. Assuming that managers pay the outside investors reservation price is a realistic assumption and consistent with many possible existing mechanisms. For example, it is equivalent to modelling the privatization procedure as a sealed-bid first price auction. In the auction, the outside investors make an offer to buy the shares. If there are infinite identical outside investors competing for these shares, their bid will be exactly their reservation price, which depends on their rational expectation of the fundamental value of the shares, based on their own information. Moreover, our results remain valid as long as the price the managers pay increases with the outsiders’ reservation price. For example, in MBOs, managers buy the shares in the market or in a tender offer from the outside investors and therefore they will pay their reservation price plus a premium.

It is therefore important to determine what is the outside investors’ reservation price. Outside investors observe the amount of restructuring $K$ undertaken in the first period and from $K$ they update their beliefs on the distribution of $\theta$. Let us define $F_K(\theta)$ the updated distribution, with expectation $E[\theta \mid K] \equiv \hat{\theta}$. On the basis of $F_K(\theta)$ the outside investors forecast that if the managers buy $\alpha_1$ shares (in addition to the $\alpha_0$ which were given free of charge or were already in their hands) the profit of the firm from period 3 onwards is $K\hat{\theta} + e(\alpha_0 + \alpha_1)$. However, outside investors do not know exactly what $\alpha_1$ is before the end of the privatization or MBO, they can only form an expectation based on $F_K(\theta)$. Thus, the outside investors’ reservation price
is given by the sum of expected future discounted profits:\(^{14}\)

\[
P^* = \frac{1}{1 - \delta} \left[ K\hat{\theta} + E(e(\alpha) \mid K) \right]. \quad (6)
\]

In other words, outside investors can compute how many shares they expect the managers to buy and therefore how much effort the managers will exert from date 3 onwards: this leads to an expectation about future profits, on which they base their reservation price. The equilibrium has to determine simultaneously the price of the shares \(P^*\) and how many shares \(\alpha_1\) a manager of a firm of type \(\theta\) will buy. In fact, \(P^*\) depends on how many shares the outside investors will expect the managers to buy. But as \(P^*\) increases, the number of shares \(\alpha_1\) a manager is expected to buy decreases (since they are more expensive).

Managers choose how many shares to buy \((\alpha_1)\) knowing the true value \(\theta\). They have to pay the price \(P^*\), minus the discount \(d \geq 0\). They maximize their payoff in period 2 with respect to \(\alpha_1\):

\[
W(\alpha) = (\alpha_0 + \alpha_1) \frac{1}{1 - \delta} [K\theta + e(\alpha_0 + \alpha_1)] - \frac{1}{1 - \delta} e(\alpha_0 + \alpha_1)^2 - (1 - d)\alpha_1 P^*. \quad (7)
\]

Note that \(e = \arg\max [(\alpha_0 + \alpha_1)(K\theta + e) - ce^2]\); therefore by the envelope theorem,

\[
\frac{dW}{d\alpha_1} = \frac{1}{1 - \delta} [K\theta + e(\alpha_0 + \alpha_1)] - (1 - d)P^* = \frac{1}{1 - \delta} \left\{ K\theta + e(\alpha_0 + \alpha_1) - (1 - d)(K\hat{\theta} + E[e(\alpha) \mid K]) \right\}. \quad (8)
\]

This is an increasing function of \(\theta\), since the expectation \(E[e(\alpha) \mid K]\) is fixed from the point of view of the managers. Managers of companies with higher \(\theta\) will want to buy more shares. For some low values of \(\theta\), managers may choose not to buy shares at all. This, in turn, affects the value of the firm, and thus \(P^*\). The solution is given by a fixed point in the following proposition.

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\(^{14}\)If we assume life terminates in period 3, the expression will be the same but without \(\frac{1}{1 - \delta}\).
Proposition 1: There exists a unique equilibrium \((P^*, \theta_0)\), where \(P^*\) is the price paid and \(\theta_0\) is a cut-off value: if the value of the firm is \(\theta \geq \theta_0\) then the manager buys \(\bar{\alpha}\) shares at the price \(P^*\); if instead the value is \(\theta < \theta_0\), he buys no shares at all. Both \(P^*\) and \(\theta_0\) are determined by the outside investors’ beliefs formed after observing \(K\) and \(P^*\) is an increasing function of \(K\).


The intuition of Proposition 1 is the following: outside investors’ reservation price depends on their expectation of the value of \(\theta\). If the true value of the firm is low relative to their expectation, the manager, who knows how much the firm is really worth, does not want to buy any shares. If instead the true value is higher than what outside investors expect, the manager wants to buy as many shares as possible. The cut-off value \(\theta_0\) depends on \(d\), \(\alpha_0\) and \(K\). The outside investors’ expectation \(E[e(\alpha)|K]\) is consistent with the actual distribution of \(\alpha = \alpha_0 + \alpha_1\): They expect the managers to acquire \(\alpha_0 + \bar{\alpha}\) shares if \(\theta \geq \theta_0\), and \(\alpha = \alpha_0\) shares otherwise.

Looking at (6) one can see that \(K\) influences \(P^*\) in two ways. The first one—the “direct effect” of \(K\) on \(P^*\)—is given by the fact that a higher \(K\) implies directly a higher value of the firm and therefore a higher \(P^*\). The second one is due to the fact that outside investors update their expectations on \(\theta\) on the basis of \(K\) and this will be reflected in the price \(P^*\) through \(\hat{\theta}\) and \(E(e(\alpha)|K)\). Both effects imply that \(P^*\) increases with \(K\).

Before moving to period 1 and solving for the optimal amount of restructuring \(K\), we introduce the following lemma, which shows that if the beliefs of the outsiders are correct, then the managers buy shares only if the discount \(d\) is strictly positive and large enough.

Lemma 1: If \(\hat{\theta} = \theta\) (the outsiders’ beliefs are correct) then the managers will never buy shares \((\alpha_1 = 0)\) if \(d = 0\).

Proof: : If \(\hat{\theta} = \theta\) then outsiders can correctly infer the number of shares \(\alpha_1\) the managers will buy and thus the price of one share is \(P^* = \frac{1}{1-\delta} [K\theta + e(\alpha_0 + \alpha_1)]\).
However, the payoff the manager receives from one share is $W(1) = \frac{1}{1-\delta}[K\theta + e(\alpha_0 + \alpha_1)] - \frac{1}{1-\delta}e[(\alpha_0 + \alpha_1)]^2$ which is always strictly lower than $P^*$. 

Q.E.D.

The intuition is immediate: the outsiders’s reservation price includes just the future profits of the company, but managers, when computing the value of a share to them, know that more equity will induce them to exert extra effort and thus internalize that extra cost. The cost of one share for managers is higher than for outside investors. Therefore, managers will buy shares either if they have better information than outside investors or if they are offered a discount. This result is not new in the literature: it is analogous, for example, to the finding for takeovers in Burkart, Gromb and Panunzi (1998) that the bidder will acquire as few shares as necessary since he does not get compensated \textit{ex ante} for the effort he will exert \textit{ex post}. The takeover literature (such as in Burkart, Gromb and Panunzi (1998)) introduces, instead of a discount, the feature that managers (or the bidder), once they have control, can appropriate of private benefits. In this way, the managers may be willing to buy shares even without a discount. To simplify the exposition, we abstract from private benefits of control for now. In Section 4.1, we show how the present model can be extended to take into account private benefits of control, and in such case a discount $d$ is no longer needed.

3.3. Restructuring

In period 1 the manager chooses the amount of restructuring $K$ that maximizes his total expected payoff at the beginning of period 1:

$$V \equiv m - cK^2 + \gamma[(\alpha_0 + \alpha_1(\theta))(e + K\theta) - ce^2] - \delta(1-d)\alpha_1(\theta)P^*, \quad (9)$$

where $\gamma \equiv \frac{\delta}{1-\delta}$ and $e$ is given in equation (5). Moreover, $\alpha_1$ is a function of $\theta$ and is equal to 0 or $\bar{\alpha}$, and $P^*$ is given by the equilibrium in the privatization game.
The manager faces the following dilemma. On the one hand, he would like to choose an investment $K$ such that he obtains (through the shares he will own) higher profits from period 3 onwards. On the other hand, undertaking a large investment requires a large effort on his part and in addition it can cause an increase in the share price.

To see what influences the choice of $K$ we identify three separate effects. Let us first assume the price $P^*$ is set exogenously. In this case, the optimal $K$ is given by:

$$K = \arg\max_K m - cK^2 + \gamma \left[ \alpha [K\theta + e(\alpha)] - ce^2(\alpha) \right]$$

which gives (using envelope theorem):

$$K(\theta) = \frac{\gamma \alpha \theta}{2c}$$

Thus, the optimal $K(\theta)$ for a given $\alpha$ is increasing with $\theta$. Notice that this is not the first best level of capital $K^{fb}$ which is given by

$$\max_K \gamma K\theta - cK^2$$

and it is equal to $K^{fb} = \frac{\theta}{2c}$. Equation (10) therefore captures the usual Jensen-Meckling (1976) effect: as the fraction of shares $\alpha$ they own increases, managers will undertake more restructuring, but as long as they do not own the entire firm, they appropriate of only part of the surplus, and thus they will underinvest.

We now identify two additional reasons why the managers may not restructure: the first one is a direct aggravation of the Jensen-Meckling effect, while the second one comes from the garbling of information. To identify the first one, let us assume that the price is not fixed, but that outside investors do not update their beliefs about $\theta$ when they observe $K$. In other words, $\hat{\theta}$ and $F_K(\theta)$ are given. Even in this case, the number of shares $\alpha$ the managers are able to buy is directly affected by the amount of restructuring $K$ they undertook in the first period. By envelope theorem, the first
order condition of the maximization of the objective function in (9) is:

\[-2cK + \gamma(\alpha_0 + \alpha_1(\theta))\theta - (1 - d)\delta\alpha_1(\theta)\frac{dP^*}{dK} = 0 \tag{12}\]

Since \(\frac{dP^*}{dK} > 0\), the optimal level of restructuring, given by the maximization of (9), is

\[K^*(\theta) = \begin{cases} 
\frac{\gamma\alpha_0\theta}{2c} & \text{if } \theta < \theta_0 \\
\frac{\gamma(\alpha_0 + \tilde{\alpha})\theta - (1 - d)\delta\bar{d}\frac{dP^*}{dK}}{2c} & \text{if } \theta \geq \theta_0 
\end{cases} \tag{13}\]

The intuition is quite simple: in Jensen-Meckling (1976) there is lack of effort because the individual who undertakes the investment appropriates of only part of the surplus. In our model this effect is even stronger because the managers not only appropriate merely a fraction \(\alpha\) of the surplus, but their net surplus (i.e. net of the price) becomes smaller as the restructuring increases. The reason is that as \(K\) increases the outside investors know that the firm is more valuable and therefore are willing to pay a higher price for the shares: as a result, the managers will never be compensated for the restructuring effort.\textsuperscript{15}

Finally, the third effect takes into account that when observing \(K\) the outside investors revise their expectations about \(\theta\). This is what happens in the reality: outside investors use whatever information they have about the recent firm history to form their opinion about the value of the firm. To study this last effect we take as the benchmark the amount of restructuring \(K^*(\theta)\), given in equation (13). This is already a sub-optimal amount of restructuring, but we will show that in equilibrium it will be even lower.

We start by looking for a fully separating equilibrium and we show that the unique separating equilibrium entails lower restructuring for all types except \(\bar{\theta}\). Therefore, in equilibrium there is always an additional welfare loss.

\textsuperscript{15}Also in a model with a wealth constraint \(w_0\), one can show that \(K^*(\theta) < \frac{\gamma\alpha_0\theta}{2c}\). In such model, the intuition of why the manager underperforms is even stronger. In fact, the larger is the restructuring undertaken by the manager in period 1, the higher is the value, and thus the price, of the firm. Thus, for a given wealth, a manager can afford to buy less shares.
Before describing the separating equilibrium, we need to note that since in a separating equilibrium outsiders’ beliefs are correct, from Lemma 1 if \( d = 0 \) managers will not buy any shares.\(^{16}\)

In Proposition 2, we characterize only the equilibrium where all managers buy shares (since it is the only interesting one). However, there always exists another equilibrium where managers never buy any shares and the amount of restructuring is \( K^*(\theta) = \frac{\gamma \alpha_0 \theta}{2c} \).

**Proposition 2:** There exists a unique separating equilibrium where all types of managers buy \( \bar{\alpha} \) shares. A manager of a type \( \bar{\theta} \) firm chooses its “second-best” level of restructuring \( K^*(\bar{\theta}) \) while managers of all other types of firms choose a level of restructuring \( K^S(\theta) < K^*(\theta) \), with \( \frac{dK^S}{d\theta} > 0 \).

**Proof:** In a separating equilibrium, necessarily \( K^S(\bar{\theta}) = K^*(\bar{\theta}) \). In fact, \( K^S(\bar{\theta}) \) is associated with the worst beliefs on the equilibrium path (from the managers’ point of view), \( \hat{\theta} = \bar{\theta} \). Therefore there is no reason for the manager of a firm of type \( \bar{\theta} \) to choose a level of restructuring different from \( K^*(\bar{\theta}) \).

A manager of a firm of type \( \theta < \bar{\theta} \), who chooses a restructuring \( K \) corresponding to a type \( \bar{\theta} \), has a payoff

\[
V(\theta, \bar{\theta}, K) \equiv m - cK^2 + \gamma[(\alpha_0 + \alpha_1)(e + K\theta) - ce^2] - \gamma(1 - d)\alpha_1[K\hat{\theta} + e]
\]

In other words, the manager knows the value of the shares depends on \( \theta \) but the price paid depends on \( \hat{\theta} \). As long as \( \alpha_1 = \bar{\alpha} \) for all \( \theta \), all the conditions given in Mailath (1987) for the existence of a unique separating equilibrium are satisfied. The condition that all types of managers buy \( \bar{\alpha} \) shares is quite intuitive: if a firm is not buying shares in the privatization process it has no reason to worry about the outside investors’ beliefs and therefore will not modify their investment to manipulate the

\(^{16}\)If we introduce private benefits of control, a positive discount \( d \) is no longer needed, see Section 4.1.
beliefs of the outside investors. From Mailath (1987) we know that $K^S(\theta)$ is given by the initial condition $K^S(\tilde{\theta}) = K^*(\tilde{\theta})$ and

$$\frac{dK^S}{d\theta} = -\frac{\partial V}{\partial \theta} \frac{\partial V}{\partial K}$$

(14)

Since $\frac{\partial V}{\partial \theta} < 0$, $\frac{dK^S}{d\theta}$ is strictly increasing and, by theorem 6 of Mailath (1987), $K^S(\theta) < K^*(\theta)$, for all $\theta < \bar{\theta}$.

Q.E.D.

Thus, in the separating equilibrium, firms undertake a sub-optimal amount of restructuring. In Figure 1 we represent the “second-best” $K^*(\theta)$ given by equation (13) and the equilibrium schedule $K^S(\theta)$. The intuition of why $K^S(\theta) < K^*(\theta)$ for all $\theta$ but $\bar{\theta}$ is the following. Assume $K^*(\theta)$ were the equilibrium schedule and take a manager of a firm of type $\bar{\theta}$: he would like to pretend to be an infinitesimally lower $\theta$ by reducing the capital investment to $K^*(\bar{\theta} - \epsilon)$. By envelope theorem, the cost of reducing the amount of capital is 0 for an infinitesimally small $\epsilon$. However, as a consequence outside investors will believe that $\theta < \bar{\theta}$ and the share price will drop, making it worthwhile for the manager to reduce $K$. The only way for a manager of a company of type $\bar{\theta} - \epsilon$ to differentiate himself from a company of type $\bar{\theta}$ is to lower even more his choice of capital (to $K^S(\bar{\theta} - \epsilon)$) so that the manager of type $\bar{\theta}$ does not find it worthwhile to imitate him anymore.

Proposition 2 is the central result of the paper. Managers are involved in a “race to the bottom”, where everybody chooses to underinvest relative to what would be its (second best) optimum ($K^S < K^*$), in order to influence the outside investors opinion about the value of the company and lower the share price. In equilibrium, they do not succeed in doing so: the investors expectations about $\theta$ are correct and the price is equal to the present value of all future expected profits. In so doing, however, managers have let pass some good investment opportunities and this implies lower profits in the future. Managers themselves would be better off if they could commit not to underinvest, but there is no such mechanism.
In the following lemma we characterize the amount of restructuring undertaken in the separating equilibrium.

**Lemma 2:** In the unique separating equilibrium, the amount of restructuring $K$ that a firm of type $\theta$ will undertake is given by the equation

$$\frac{2c\eta}{\eta - \xi} K = AK^\frac{\xi}{\eta} - \eta \theta$$

where $\xi \equiv \gamma \alpha - \gamma (1 - d) \bar{\alpha}$, $\eta \equiv \gamma (1 - d) \bar{\alpha}$ and $A$ is a constant of integration such that

$$A \equiv \left[ \frac{\xi \bar{\theta}}{2c} \right]^{\frac{\xi}{\eta}} \frac{\eta^2}{\eta - \xi}.$$

**Proof:** From equation (14), we can write the differential equation $-2cKdK + \xi \theta dK - \eta K d\theta = 0$. Solving the differential equation we obtain

$$-2cKdK + \frac{dK^{\xi - \eta - 1}}{K^{\xi - 1} \theta^{-\eta - 1}} = 0.$$ 

If we define $h \equiv K^{\xi - \eta}$ then the expression above becomes

$$-2cK^{\xi - \eta - 1} dK + dh = 0$$

But $K^{\xi - \eta - 1} = h = K^{-\frac{\xi}{\eta}} h^{1+\frac{1}{\eta}}$. The differential equation therefore becomes

$$\frac{2c}{\eta - \xi} dK^{-\frac{\xi}{\eta} + 1} = -dh^{-\frac{1}{\eta}}.$$ 

The solution to this differential equation is the one given in the lemma. To determine the constant of integration we use the fact that $K^S(\tilde{\theta}) = \arg\max V(\tilde{\theta}, \tilde{\theta}, K) = \frac{\xi \bar{\theta}}{2c}$.

Q.E.D.

Let us now see what happens to the equilibrium as $\alpha_0$ and $d$ change. These measures are often seen as unfair advantages offered to the managers. We are going
to show that an increase in the number of shares the managers obtain for free or in the
discount is actually going to be welfare improving, since the managers will undertake
more restructuring.

**Lemma 3:** When \( \alpha_0 \) increases, \( P^* \), \( K^*(\theta) \) and \( K^S(\theta) \) increase (for any \( \theta \)). When \( d \)
increases, \( K^*(\theta) \) and \( K^S(\theta) \) increase (for any \( \theta \)) but \( P^* \) does not change.

**Proof:** See Appendix A2.

The intuition of this result is quite simple. If the managers own (or are given for
free) a large number of shares \( \alpha_0 \), they will have more incentives in maximizing the
value of these shares and therefore the optimal amount of restructuring \( K^* \) undertaken
in period 1 increases. This is observable to everybody and therefore the share price \( P^* \)
increases to reflect the higher value of the shares. Since the asymmetry of information
has not been made worse, the managers incentive to misrepresent the type has not
increased and the amount \( K^S \) also increases.

Similarly, when the discount \( d \) increases the cost to the managers of increasing
\( K \) (i.e. paying a higher price) are reduced and therefore they will be willing to
exert a higher effort to have a higher \( K \). Thus the equilibrium changes and the
underinvestment problem is reduced.

We now characterize the pooling equilibria, where managers of firms of every type
\( \theta \) choose to undertake the same level of restructuring \( K_P \). In equilibrium, \( K(\theta) = K_P, \)
\( \forall \theta \), therefore the outside investors cannot update their beliefs when they observe \( K_P \)
and \( F_{K_P}(\theta) = F(\theta) \). Before doing this, however, we should point out that, as it will
be clear from the construction of the pooling equilibria in the Appendix, any pooling
equilibrium will require very strong assumption about the out-of-equilibrium beliefs.
In fact, if the manager of a firm of type \( \theta \) chooses to pool, this implies a larger
investment \( K \) and thus a larger effort than he would have undertaken otherwise.
In addition, the investors’ beliefs will be that \( \hat{\theta} = E(\theta) > \bar{\theta} \), which implies more
expensive shares. Why would then he want to pool? It will be optimal for him to
do so only if the alternative is to be considered of very high \( \theta \) (making the shares
very expensive to buy). It can therefore be shown that if a refinement such as the Grossman and Perry (1986) is used, only the separating equilibrium will remain. For completeness, we present in this paper both the pooling and separating equilibria, but in the general discussion we insist more on the separating equilibrium, on the ground that it is much more robust and therefore more likely to arise.

In the next proposition, we show that pooling equilibria are possible only if the discount \( d \) is sufficiently high.\(^{17}\) This condition does not follow from Lemma 1, since in a pooling equilibrium the outsiders’ beliefs are not correct. The intuition is the following. In a pooling equilibrium outside investors cannot distinguish between different types and the price of the shares \( P^* \) will depend on the average \( \theta \) and thus will be too high for managers of low \( \theta \) firms. If the discount \( d \) is not sufficiently large, they will not buy any shares. But if they do not buy any shares, they have no incentives to garble the information and to choose \( K_P \). Thus it is necessary to have a minimum discount \( d \) which induces the managers of low \( \theta \) firms to buy overpriced shares.

**Lemma 4:** If the discount \( d \) is not sufficiently high, there exists no pooling equilibrium.

**Proof:** First of all, notice that a manager of a firm of type \( \theta \) chooses a level of restructuring \( K_P \) only if it expects to buy \( \bar{\alpha} \) shares in period 2. In fact, if the firm is not going to buy shares in the privatization process, it has no interest in garbling information, and it chooses \( K = \frac{\gamma_{\text{profit}}}{2c} \) whatever are the beliefs associated with it. Assume now that \( K_P \) is an equilibrium and all firms buy \( \bar{\alpha} \) shares. This is the same as requesting that the cut-off value \( \theta_0 \leq \theta \), where the cut-off value is given by (using the expression for \( \theta_0 \) in the Appendix A1 and substituting for \( P^* \) from equation (6):

\[
\theta_0 = \frac{1}{K_P} \left[ (1-\delta)(1-d)P^* - \frac{\bar{\alpha} + 2\alpha_0}{4c} \right] = \\
= \frac{1}{K_P} \left[ (1-d) \left( K_P E(\theta) + \frac{\bar{\alpha} + \alpha_0}{2c} \right) - \frac{\bar{\alpha} + 2\alpha_0}{4c} \right]
\]

\(^{17}\)Again, this is true only if there are no private benefits of control, see Section 4.1.
For the cut off value to be at least equal to $\bar{\theta}$ it is necessary that

$$\theta \geq (1 - d)E(\theta) + \frac{1}{K_P} \left[ (1 - d) \frac{\bar{\alpha} + \alpha_0}{2c} - \bar{\alpha} + 2\alpha_0 \right].$$

Let us take $d = 0$. Then the inequality becomes: $\theta \geq E(\theta) + \frac{1}{K_P} \bar{\alpha}$. This is clearly not possible and therefore it implies that if $d$ is not strictly positive at least some types of managers will not buy shares, which is inconsistent with the pooling equilibrium.

Q.E.D.

We can now present the range of restructuring $K$ that can be sustained in a pooling equilibrium and establish its lower and upper bound.

**Proposition 3:** In any pooling equilibrium where all types of managers choose the same amount of restructuring $K_P$, we have


2. $K_P \leq K^*(\theta_E)$ if $\theta \leq \frac{1}{2} [(1 - d)\bar{\alpha}\theta_E + \alpha\theta_E]$ or $\bar{\alpha} \geq \frac{\alpha(\theta_E - \theta_P)}{(1 - d)\theta_E}$.

**Proof:** See Appendix A3.

Point (1) shows that managers of low $\theta$ firms will not restructure above a certain level and managers of high $\theta$ firms will not reduce their restructuring more than a certain amount. In other words, managers of low $\theta$ firms restructure too much and managers of high $\theta$ firms restructure too little. This implies that even when we do observe restructuring being undertaken, this may still be suboptimal, as it is undertaken by the wrong firms.

Point (2) shows that when the lowest type $\bar{\theta}$ is very low the pooling equilibrium implies a restructuring lower than the “second best” restructuring $K^*$ that would have been chosen by the average type $\theta_E$. That is because the manager of type $\bar{\theta}$ will accept to undertake only low levels of restructuring and everybody else will have
to pool with him. The same will happen when the maximum number of shares \( \tilde{\alpha} \) that can be bought is high, because in such case the incentive to manipulate outside investors’ expectations is also high and all types of managers have an incentive to pool on low values.

Finally, the following lemma looks at how the equilibrium changes when \( d \) or \( \tilde{\alpha} \) change.

**Lemma 5:** The upper bound of \( K^p, \tilde{K} \), is such that \( \tilde{K} - K^*(\theta_E) \) decreases when \( d \) or \( \tilde{\alpha} \) increase.

**Proof:** See Appendix A4

The intuition is similar to the one of Proposition 3: the higher the discount or the number of shares managers can buy, the higher are the incentives to manipulate investors beliefs and therefore to underperform.

### 4. Extensions

As already mentioned, the main conclusions of the paper remain true also when we change the model in various directions. We therefore now briefly illustrate several extensions that do not change the main conclusions.

#### 4.1. Private benefits of control

Our framework allows us to consider the possibility of private benefits of control. In other words, we can assume that if a manager owns at least \( \alpha \) shares, he has control of the firm and has benefit of control \( B \), so that his utility from period 3 onward will be equal to

\[
U = \alpha \pi - ce^t + BI\{\alpha \geq \tilde{\alpha}\},
\]  

(15)
where \( I\{\alpha \geq \bar{\alpha}\} \) is an indicator function that takes value 1 if \( \alpha \geq \bar{\alpha} \). The effort maximization in each period (starting from period 3) does not change. In period 2, when the manager chooses whether to buy shares or not, his utility function is

\[
W(\alpha) = (\alpha_0 + \alpha_1) \frac{1}{1 - \delta} [K\theta + e(\alpha_0 + \alpha_1)] - \frac{1}{1 - \delta} c[e(\alpha_0 + \alpha_1)]^2 - (1 - d)\alpha_1 P^* + \frac{1}{1 - \delta} BI\{\alpha \geq \bar{\alpha}\}.
\]

(16)

If \( \alpha_0 < \bar{\alpha} \) (which is realistic), then it is easy to prove that even if the discount \( d \) is equal to zero there exists a unique equilibrium with a price \( P^* \) where managers will buy either no shares or the maximum \( \bar{\alpha} \). In this case, however, \( \bar{\alpha} \) is determined so that \( \bar{\alpha} + \alpha_0 = \bar{\alpha} \). In fact, if there is no discount, managers have no interest in buying more than \( \alpha \), i.e. the minimum shares necessary to have control. Managers will buy shares even with no discount because in exchange they will receive private benefits \( B \) and therefore they will be willing to pay the reservation price of outside investors.

If we now look at the restructuring choice in period 1, there still is a unique separating equilibrium where managers undertake less restructuring. Similarly, the pooling equilibria we characterized in Proposition 3 do not change. In other words, introducing private benefits of control increases the managers’ incentives to buy shares, but the incentives not to restructure in order to reduce the price remain.\(^{19}\)

One may wonder whether it is true that managers may not be interested in obtaining additional shares once they have the minimum necessary to have control. However, evidence given by Boycko, Shleifer and Vishny (1994) and Filatotchev et al. (1999) shows that this is not true in reality, since managers are eager to buy shares even after they already have control. This is consistent with Burkart, Gromb and Panunzi (1998), who argue that benefits of control increase with the control stake

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\(^{18}\)They may still require a positive discount \( B \) if the benefit is small.

\(^{19}\)In this context, we can also see what happens if a manager already owns \( \alpha_0 \) shares, but is not part of the MBO group bidding for control: in such case the manager will not appropriate of any private benefits of control. Therefore, since the price is the correct price, the manager will have an incentive to sell rather than buy shares.
Moreover, if managers are not sure to obtain control, they could have even more incentives to underperform. This is considered in the next section.

4.2. Alternative bidders for control of the firm

One may argue that if managers seriously underperform and delay restructuring in period 1, disciplinary takeovers may take place. To a certain extent this may be true and it may impose a limit to how much a manager can underinvest in the company. However, the possibility of alternative bidders interested in taking over the firm may actually aggravate the perverse effects we have been focusing on. In fact, alternative bidders are likely to have imperfect information about the value of the firm and they will have to form expectations about its value in order to decide how much to bid. Therefore, alternative potential large shareholders will look at the level of restructuring $K$ undertaken by the firm in the first period and will use such observation to update their information on $\theta$. Although the actual bid will depend on what they think the value of the firm will be under their control, it is likely to depend also on the expected $\theta$. The same perverse effects stressed in the main part of the paper (where managers were concerned about the reservation value of outside investors) arise when managers are concerned about alternative bidders and how much they are willing to pay. Therefore, if managers plan to have an MBO in the future, the possibility of disciplinary takeovers or of other investors interested in acquiring control of the company may very well increase their incentives to underinvest and delay restructuring rather than reducing them.

\footnote{This is similar in spirit to what is happening at present at the Italian airline company Alitalia where, following a privatization announcement, employees (thus not only managers) have gone on strike several times and blocked several restructuring plans, in order to discourage potential bidders who aim at cutting labour costs after buying Alitalia. The aim is to make Alitalia to look as bad as possible and discourage bidders who think they can make it profitable. See “Will anyone offer to buy Italy’s ailing national airline?”, \textit{The Economist} January 4th 2007 or “Unions prepare for battle over Alitalia privatisation plans”, \textit{Financial Times} January 11th 2007.}
4.3. Performance-contingent remuneration in the first period

In the main part of the paper we assumed that the manager in period 1 was paid a fixed salary $m$ (equation (1)). More generally, one can imagine that in the first period the manager may also be paid contingent on his performance in period 1 (i.e. on the profits in period 1). We argued then that this assumption would actually strengthen our result and here we briefly illustrate how. The profits in period 1 will be equal to

$$\pi_1 = \pi(\theta) - C(K) + e_1$$

where $\pi(\theta)$ are the basic profits, function of the “quality” of the company, as captured by the parameter $\theta$. In general, the way these profits depend on $\theta$ will be different than in the subsequent periods, since at this stage the restructuring is only under way. The fact that the restructuring, or investment, $K$ is undertaken in this period implies that the firm is bearing a cost $C(K)$. This is the monetary cost of the investment or restructuring and is different from the cost that the manager bears, which is just the private utility cost of undertaking the effort of the restructuring. Finally, the profits depend on the effort $e_1$ exerted by the manager in that same period (this justifies why the remuneration is contingent on the performance). The payoff of the manager in period 1 will be:

$$U_1(e_1, K) = \beta \pi_1 - ce_1^2 - cK^2 = \beta(\pi(\theta) - C(K)) - ce_1^2 - cK^2$$

where $\beta > 0$ and $\beta \pi_1$ is the compensation on the basis of period 1 profits (more generally, the manager compensation will be an increasing function of $\pi_1$).

Then the total expected payoff of the managers at the beginning of period 1, previously given in equation (9), becomes

$$V \equiv \beta[\pi(\theta) - C(K)] - ce_1^2 - cK^2 + \gamma[(\alpha_0 + \alpha_1(\theta))(e + K\theta) - ce_1^2] - \delta(1 - d)\alpha_1(\theta)P^*.$$

The first order conditions with respect to $K$ (taking the price $P^*$ for given) will
be

\[-C'(K) + \gamma \alpha \theta - 2cK \leq 0\]

and it is thus clear that the optimal \(K(\theta)\) is thus lower than \(\frac{\gamma \alpha \theta}{2c}\) (given in equation (11)). Consequently, the second best \(K^*(\theta)\) and the separating equilibrium \(K^S(\theta)\) will also be lower by the same amount. Thus the underinvestment result highlighted in the paper is strengthened.

### 4.4. Management turnover

In the previous sections we assumed that managers remain in control after a privatization or an MBO. In the case of an MBO, by definition the managers will still be in control after the MBO.\(^{21}\) In the case of a privatization, however, there may be a management change. For the purpose of this paper, it is crucial when the management change takes place. If the old managers remain in control for all period 1 and are substituted only when the privatization takes place, then the incentives to delay restructuring remain. Even if they were sure they will not remain in control, managers still have an incentive to buy shares if they know that the firm has a high \(\theta\).

The payoff in (9) remains the same, only without the cost of effort \(ce^2\), and the same results of underperforming during to signal jamming remain. The main difference is that a positive discount is no longer necessary (since managers are now just like outside investors but with better information). However, if managers are replaced at the beginning of period 1, then they will not be able to choose \(K\) and delay restructuring. In Section 6 we mention empirical evidence from Lopez-de-Silanes (1997) that replacing managers with a professional “privatizer” at the beginning of period 1 increases the value of the company at the privatization stage.

\(^{21}\)It could happen that management started an MBO, but another bidder arrived and won the takeover contest. As long as in period 1 there is a positive probability for the MBO to go through, the \textit{ex ante} perverse effects arise, and may be aggravated by the fear to lose control, as argued in the previous section.
4.5. Different methods of acquiring shares

There are numerous methods managers can use to increase their stake in the company. Which method is relevant depends also on whether the event in period 2 is an MBO or a privatization. In the main part of the paper we assume that managers pay the outside investors’ reservation price (in the case of privatization, for example, this would correspond to using a first-price sealed bid auction). We already argued then that our results remain true as long as the price paid by the managers is an increasing function of outside investors’ reservation price. This assumption would capture the essence of most of the methods used in MBOs or privatizations. For example in MBOs, managers usually buy shares in the market or through a tender offer at a premium: as long as the premium is proportional to the share price or increasing with it, the results will all go through.

In this section, we briefly consider two additional possible methods. First of all, privatization may take place through vouchers. Second, management can buy the shares through a direct sale from either the state (in the case of a privatization) or the company itself (in the case of a divisional MBO).

Let us start by considering vouchers’ privatization: each individual (both managers and outside investors) receives a given number of vouchers, which can be used to buy shares in any of the firms undergoing a privatization. The number of shares bought by one voucher depends on the total number of vouchers individuals offer in exchange for those shares, so that supply equals demand. If managers use $x$ vouchers to obtain shares in the firm, they will obtain a fraction of the firm given by

$$\alpha_1 = \frac{x}{x + n(K, \hat{\theta})}$$

where $n(K, \hat{\theta})$ is the demand (in terms of number of vouchers) for the firm’s shares from outside investors, which depends on the expected value of $\theta$ and the observed restructuring $K$. Since $\frac{dn}{dK}$ and $\frac{dn}{d\hat{\theta}}$ are positive (outside investors will use their vouchers with the firms they consider more valuable), then $\frac{d\alpha_1}{dK}$ and $\frac{d\alpha_1}{d\hat{\theta}}$ are negative. Therefore,
the managers’ objective function is

\[ m - C(K) + \gamma(\alpha_0 + \alpha_1(K, \hat{\theta}))\pi \]

and the model is very similar to the one in the previous section.

Finally, the case of direct sale could be modelled as a bargaining between the government (or the conglomerate selling the division) and the managers, where managers have better information about the value of the firm and have incentive to underperform and garble information before they start to bargain. The same results as in the main part of the paper would carry through.

4.6. Restructuring after the privatization or the MBO

Our model also allows for managers to restructure after the privatization or the MBO, if they did not do it before. In reality, delaying restructuring can be more costly for the firm than early restructuring due to lost opportunities in a changing market environment. Therefore, the objective function could be, for example, \( W - cK^2 + \gamma[\alpha(\bar{K}\theta - 2cK^2]] \). Thus, managers can conduct the restructuring later, but at double the marginal cost. With such a set-up, the firm still wants to delay restructuring for fear of revealing the true value of the firm and obtaining less shares subsequently. Our result is strengthened, since delaying restructuring becomes less costly.

5. Empirical evidence on MBOs

Following an MBO, the fraction of the company owned by management increases dramatically and it is often argued that this increases the managers’ incentives to improve performance. A rational for going-private buy-outs is to be able to offer proper incentives to managers: If the restructuring effort managers have to undertake is very large, only a disproportionate share of the company will provide appropriate incentives. Since shareholders may object to managerial compensation if it is too
large, an MBO may be the appropriate solution (see Weston, Mitchell and Mulherin, 2004). Kaplan (1989) reports management ownership before and after a buy-out. The median ownership of all managers increases from 5.88% to 22.63%.

The empirical evidence shows that performance improves after an MBO. Muscarella and Vetsuypens (1990) show that more than two thirds of the firms in their sample disclose at least one restructuring activity undertaken since the buy-out; as a result, the firms realized substantial improvement in operating performance. Similarly, Kaplan (1989) and Smith (1990) show improvement in performance. These last two papers also argue that the improvement in performance cannot be due to asymmetry of information. In particular, Kaplan (1989) shows that managers and directors who were not part of the management doing the buy-out actively sold their shares. This is inconsistent with the hypothesis that the shares were undervalued. Note that this evidence is perfectly consistent (and actually supports) with the hypothesis put forward in this paper. In fact, although managers start by having better information about the value of the firm, by the time the MBO takes place, outside investors are perfectly aware of the value of the shares and the price is the fair price. Unfortunately, the damage (i.e. the lack of restructuring) has already taken place. Moreover, it is an implication of this model that managers and directors who are going to remain in the company but are not going to be part of the controlling group will want to sell their shares, unless they could buy them at a discount.

Ofek (1994) also rejects the private information hypothesis looking at the outcome of unsuccessful MBOs. He finds that unsuccessful MBOs do not show a performance improvement (this is also consistent with the model in this paper). He also finds that in 47% of the cases the management was substituted after an unsuccessful MBO, and in most of the remaining cases the company was taken over. While not directly implied by our model, this findings are consistent with the possibility that after the failure of the MBO it became more clear that the manager had underinvested or in some way decreased the value of the firm.

Interestingly, there is empirical evidence that the prospective of a buy-out of a
company or division induces managers to manipulate earnings downwards. De Angelo (1986) first suggested it, but found no support for it, but two more recent studies have found support for this hypothesis. Perry and Williams (1994) find evidence that managers understate earnings before an MBO. Similarly, Marquardt and Wiedman (2002) find evidence that before a buy-out, managers delay revenue recognition. This is very relevant for our study, since the incentives that induce managers to understate earnings (i.e., in order to buy shares at a lower price) are exactly the same that would induce managers to underinvest or delay restructuring.

6. Empirical Observations of State Firms before Privatization

The paper that most closely tests the implications of our model is Lopez-de-Silanes (1997). He studies 236 privatizations in Mexico between 1983 and 1992 and looks at the determinants of privatization prices. He finds that privatizations with shorter sale times fetch better prices, where he defines sale time as the period between the first rumor of privatization and the announcement of the winner (one extra year of sale time reduces the net privatization price by 36%). This is exactly what is predicted by the model in this paper: once the possibility of a privatization becomes known, managers have an incentive to postpone restructuring and underinvest. The longer is the period in which they have an incentive to postpone restructuring (in other words the longer is the period 1 of the model) the larger is the decrease in the value of the firm.

Moreover, Lopez-de-Silanes (1997) finds that if the managers are replaced at the onset of the privatization process by a professional “privatizer”, whose task is to clean up the company and to get it ready to be sold as soon as possible, the privatization price is higher. Consistently with the hypothesis in this paper, the “privatizer” does not have the perverse incentives of the management and therefore all the inefficiency of period 1 is avoided.

Additional supporting empirical and anecdotal evidence comes from the recent
wave of privatization in transition economies. The problems stressed in this paper are particularly acute in these emerging markets for three reasons. First of all, there is a particular need for restructuring and investment in human capital. Second, the value of the firms in transition economies (before they are privatized) is less transparent and it is therefore easier for managers to disguise (or at least try to disguise) temporarily the true potential of a company in order to buy it on the cheap. Finally, in these economies there is usually a lack of institutional investors who can act as large shareholders and therefore it is important to encourage accumulation of shares by insiders (such as managers and workers) by favoring them in the privatization phase.

Managers of state firms are crucial players during the transition, since they have *de facto* control of state firms before they are privatized. Therefore, they can decide how much to restructure the company. Several empirical works verified this point. For example, Fan and Shaffer (1993) documented managers’ strong control of Russian state firms before privatization. Perhaps, the most illustrating example of managers’ power and incentive is the so-called spontaneous privatization, which is a popular strategy used by managers during the early days of transition (see Kornai, 1992; Hillman, 1992; and Shleifer and Vishny, 1992). In spontaneous privatizations, managers (or, more generally, the insiders) make under-the-table deals with outside investors by selling the firms’ assets at below-market prices. Later, the outside investors pay back the insiders one way or another. This is an extreme form of underinvestment (where $K$ is negative).

The key argument of our paper is that managers of state firms act strategically in order to position themselves properly for the coming privatization and this effect may offset positive incentives of restructuring. Indeed, this has been the focus of a large group of empirical works. A very revealing case regarding managers’ incentives is the massive break-ups of state firms since the start of transition. In Czech Republic, as a result of the break-ups, the number of state firms increased from 700 in 1990 to 2,000 in 1992. Lizal, Singer, and Svejnar (1995) is a very careful study of the rationale
behind these break-ups. They found that break-ups cannot be explained by pure value maximizing motives. To the contrary, the break-ups are mostly value decreasing and are driven by managers’ strategic consideration for the coming privatization. Kotrba (1995) explicitly analyzes the incentive of state firm managers before privatization. He listed seven commonly observed moves by managers before privatization. All of these strategies, such as “making privatization as lengthy as possible and using the lack of ownership control to support private activities,” are clearly counter-productive from the restructuring point of view.

As a comprehensive analysis of the enterprise behavior before privatization, Estrin, Gelb, and Singh (1995) conduct a unique comparative study of state firms across the former Czech and Slovak Republic, Hungary, and Poland. They base their analyses on 43 case studies of state firms in these economies, which adopted rather different approaches to enterprise reform. As a major conclusion, they found that whether or not a state firm had a long-run strategy of restructuring depended crucially on the extent of ownership change. In other words, managers of firms yet to be privatized are unlikely to take initiatives to form comprehensive plans to increase efficiency.

Besides these systematic empirical works, there are plenty of anecdotal stories and case studies about managers’ maneuvers before privatization. For example, before a state firm in the Czech Republic were privatized, the incumbent managers proposed a management buy-out. In order to minimize the price to pay, the managers tried very hard to demonstrate that the firm was worthless. One of their strategies was to order a large number of productive workers to do nothing but washing windows for one year. In a case study by Edison (1994), a well-known formerly East German optical company was partially purchased by a West German firm. However, the privatization process was not completed, since a significant amount of shares were still controlled by the government. As a result, the Eastern and Western management team colluded and intentionally delayed actual restructuring of the firm. Their purpose was clearly

22This is based on a field trip experience of Josef Kortba. See Kortba (1995) for systematic analysis.
to show that the firm was worthless so that the Western company could buy the remaining shares at a very low price, which was what actually happened.

7. Conclusions and Further Discussions

The main result of the paper is that providing managers with the correct \textit{ex post} incentives may distort \textit{ex ante} incentives. This is particularly important for events where managers can increase substantially their ownership and where restructuring is particularly crucial for the value of the firm, as in privatizations and MBOs. Bringing the argument to the extreme, one can argue that while usually the literature argues that MBOs and privatizations take place because the firm needs restructuring, the need for restructuring may have arisen exactly because the managers were expecting an MBO or privatization to take place.

Moreover, note that the effect in our model does not arise because managers do buy underpriced shares: although asymmetry of information plays a role in our model (since managers want to bias outside investors expectation), this happens before the MBO or privatization: by the time these events take place everybody knows the true value of the firm, but the damage to the value of the firm (because of the delayed restructuring) has already taken place.

The results in this paper do not imply that managerial ownership should not be encouraged, but rather that more attention should be paid to the period before managerial ownership is increased, and to the way it is increased. For example, in the context of a privatization, the incentives to underinvest would be reduced if managers were compensated contingent on the revenues of the privatization. Also, managerial compensation should be made, if possible, contingent on $K$.

Finally, an implication of this paper is that it is not obvious what causes the increase in productivity following MBOs and privatizations. At least in part, such increase could be due to underperformance in the previous period.
Appendix

A1. Proof of Proposition 1

The manager chooses $\alpha_1$ in order to maximize equation (7) subject to the contrainst that $0 \leq \alpha_1 \leq \bar{\alpha}$. The solution to such maximization depends on the value of $\theta$. If $\theta$ is high enough, the first order condition in equation (8) is always strictly positive, and the solution is $\alpha_1 = \bar{\alpha}$. If $\theta$ is low, the function $W(\alpha)$ is convex and the solution is one of the two extremes: $\alpha_0$ or $\bar{\alpha}$. As $\theta$ increases, $W(\alpha_0 + \bar{\alpha})$ increases more than $W(\alpha_0)$, therefore there exists a unique cut-off value $\theta_0$ such that the insider with value $\theta_0$ is indifferent between buying and not buying: $W(\alpha_0 + \bar{\alpha}) = W(\alpha_0) = \frac{1}{4c} \left[ \alpha_0 K \theta + \frac{\bar{\alpha}^2}{4c} \right]$. If $\theta \geq \theta_0$, the solution is $\alpha_1 = \bar{\alpha}$, if $\theta < \theta_0$ the solution is $\alpha_1 = 0$. Such cut-off value is $\theta_0 = \frac{\bar{\alpha}}{K} \left[ (1-d)(1-\delta)P^* - \frac{1}{4c} (\bar{\alpha} + 2\alpha_0) \right]$.

Finally we can show that also $P^*$ is uniquely determined, given $F_K(\theta)$. In fact, the equilibrium price is given by equation (6), which can be rewritten as

$$ (1-\delta)P^* = K \hat{\theta} + \frac{\alpha_0}{2c} + \frac{\bar{\alpha}}{2c} Pr\{\theta \geq \theta_0\} $$

(17)

where $Pr\{\theta \geq \theta_0 \mid K\} + Pr\{\theta \geq g(P^*) \mid K\}$ is computed according to the updated distribution $F_K(\theta)$.

Since $\frac{d\theta}{dP^*} = g'(P^*) > 0$, the function $g(P^*)$ can be inverted and the solution to equation (17) is given by:

$$ R(P^*) = (1-\delta)P^* - \frac{\bar{\alpha}}{2c} Pr\{P^* \leq g^{-1}(\theta)\} - K \hat{\theta} - \frac{\alpha_0}{2c} = 0 $$

(18)

Since $Pr\{P^* \leq g^{-1}(\theta)\}$ decreases as $P^*$ increases, we can conclude that $R'(P^*) > 0$, $R(0) < 0$ and $R(+\infty) = +\infty$. Therefore there exists a unique solution $P^*$ for each $K$.

Equation (18) defines a unique solution $P^*(\alpha_0, d, K)$. Given such function, we can substitute it in the expression for $\theta_0 = g(P^*)$. Finally, the optimal function $\alpha_1(\alpha_0, d, K)$ is given by:

$$ \alpha_1 = \begin{cases} \bar{\alpha} & \text{if } \theta \geq \theta_0 \\ 0 & \text{if } \theta < \theta_0 \end{cases} $$

(19)

---

23 If $\bar{\alpha}$ were endogenous, as in a model with a wealth constraint, such constraint would be replaced by the budget constraint that $0 \leq \alpha_1 \leq w_0 (1-d)P^*$.

24 The same logic, with slightly more complex computations, applies in the case of a model with wealth constraint.
A2. Proof of Lemma 3

Let us first look at what happens when $\alpha_0$ increases. Notice that in a separating equilibrium $\theta_0 \leq \theta$ necessarily (i.e. all managers buy shares), therefore we need to check that $\theta_0$ does not increase (otherwise we may need a higher $d$ for the equilibrium to exist). From the expression of $\theta_0$ given in Appendix 1 one calculate that

$$\frac{d\theta_0}{d\alpha_0} = \frac{1}{K} \left[ -\frac{1}{2c} + (1-d)(1-\delta) \frac{\partial P^*}{\partial \alpha_0} \right].$$

The first term is negative, to determine the sign of the second term we have to look at how $P^*$ changes when $\alpha_0$ changes. From equation (6) one can see that the derivative is $\frac{\partial P^*}{\partial \alpha_0} = \frac{1}{2c(1-\delta)} > 0$, i.e. $P^*$ increases. Therefore

$$\frac{d\theta_0}{d\alpha_0} = \frac{1}{K} \left[ -\frac{1}{2c} + 1 - d \right].$$

Therefore, if $d = 0$ $\theta_0$ does not change and $P^*$ increases. If instead $d > 0$, $\theta_0$ decreases, which will make $P^*$ increase even further (but not enough to cancel the effect on $\theta_0$). This is however all assuming no change in the amount of restructuring $K$. Let us now see what happens to $K$.

Let us start by looking at the optimum $K^*(\theta)$ given in equation (13). Notice that only the case $\theta \geq \theta_0$ is relevant in a separating equilibrium. Therefore,

$$\frac{dK^*}{d\alpha_0} = \frac{1}{2c} \left[ \gamma \theta - (1-d) \delta \alpha \frac{\partial^2 P^*}{\partial K \partial \alpha_0} \right].$$

From equation (6), $\frac{dP^*}{dK} = \frac{\theta}{1-\delta} + \frac{1}{2c(1-\delta)} \frac{dE(\alpha_1|K)}{dK}$. But we know that in a separating equilibrium $\alpha_1 = \bar{\alpha}$ $\forall \theta$. Thus the second term is equal to 0 and the derivative of $\frac{dP^*}{dK}$ with respect to $d$ is equal to 0. Therefore, $\frac{dK^*}{d\alpha_0} = \frac{\gamma \theta}{2c}$ which means that when $\alpha_0$ increases also $K^*$ increases.

Finally, we have to show what happens to the restructuring choice $K^S$ in the separating equilibrium. First of all, since $K^S(\overline{\theta}) = K^*(\overline{\theta})$, $K^S(\overline{\theta})$ increases. To see what happens for the lower values of $\theta$ notice that from Mailath (1987), as shown in the Proof of Proposition 2, we know that

$$\frac{dK^S}{d\theta} = -\frac{\partial V}{\partial \theta} = -\frac{\gamma (1-d) \pi K}{-2cK + \gamma (\alpha_0 + \bar{\alpha}) \theta - \gamma (1-d) \pi \theta}.$$  (20)
It is easy to see that as $\alpha_0$ increases the denominator of this fraction becomes smaller while the numerator does not change. Thus the derivative (which we showed before is positive) decreases and the curve is flatter. Therefore, the curve representing $K_S(\theta)$ is higher at the highest level $\bar{\theta}$ and it is flatter: that means it is higher for any $\theta$, i.e. $K_S(\theta)$ increases when $\alpha_0$ increases. (Notice that if $K$ increases that is one additional reason why $\theta_0$ decreases.

Let us now look at what happens when $d$ increases. First of all, from equation (6) we can see that as $d$ increases $P^*$ does not change. If we then look at $\theta_0$ we see that $\frac{d\theta_0}{dt} = -\frac{1}{K}(1 - \delta)P^* < 0$.

We conclude by look at what happens to $K^S$. Since $K^S(\bar{\theta}) = K^*(\bar{\theta})$, $K^S(\bar{\theta})$ increases. Moreover, we can see what happens to $\frac{dK^S}{dt}$ when $d$ changes. Looking at equation (20), we can see that as $d$ increases, the numerator of the right hand side of the equation decreases, and the denominator increases. Thus, as $d$ increases, $\forall \theta$, $\frac{dK^S}{dt}$ increases. But that means that the highest point of the function $K^S(\theta)$, $K^S(\bar{\theta})$ is higher and the slope of this function, as it increases from $K^S(\theta)$ to $K^S(\bar{\theta})$, is flatter. This implies that necessarily $K^S(\theta)$ is higher $\forall \theta$.

Q.E.D.

A3. Proof of Proposition 3

Let us define $K_P$ as the level of restructuring that takes place in a pooling equilibrium. Since all types of managers are choosing this restructuring, in equilibrium the outside investors’ belief is $E(\theta) \equiv \theta_E$. We now look at the incentives to deviate of types $\theta$ and $\bar{\theta}$. (Although it is not important for the present proof, it can be shown that these are actually the only types that need to be checked in equilibrium).

We check the equilibrium with the strongest out-of-equilibrium beliefs, i.e. we assume that out-of-equilibrium the outside investors believe that the type is $\bar{\theta}$. For any other belief, it will be harder to sustain the equilibrium, so the set of restructuring choices that can arise in a pooling equilibrium will be smaller. We also assume that when types $\theta$ or $\bar{\theta}$ deviate they still want to buy shares in period 2, so that the shares that they will have ex post, $\alpha_0 + \bar{\alpha}$ are the same (the same analysis can be conducted if the type, when he deviates, will not buy shares any more).

$K_P$ is an equilibrium if and only if managers of type $\theta$ do not want to deviate, i.e. if and only if:

$$V(\theta, \theta_E, K_P) \geq V(\theta, \bar{\theta}, K(\theta, \bar{\theta}))$$

(21)

where

$$K(\theta_1, \theta_2) \equiv argmax_KV(\theta_1, \theta_2, K) = \frac{\gamma[\alpha \theta_1 - (1 - d)\bar{\alpha} \theta_2]}{2c}$$

is the amount of restructuring chosen by a type $\theta_1$ if the outside investors believe he is of type $\theta_2$. 

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Using the definition of $V$, condition (21) can be rewritten as (where the effort $e$ has been simplified, due to the fact that as the number of shares does not change, neither does the effort):

$$-cK_p^2 + cK^2(\theta, \overline{\theta}) + \gamma K_P[\alpha\theta - (1 - d)\tilde{\alpha}\theta_E] - \gamma K(\theta, \overline{\theta})[\alpha\theta - (1 - d)\tilde{\alpha}\overline{\theta}] \geq 0$$

which becomes, using the definition of $K(\theta_1, \theta_2)$ given above:

$$K_p^2 + K^2(\theta, \overline{\theta}) - 2K_PK(\theta, \theta_E) \leq 0$$

The solution to this inequality is

$$K \equiv K(\theta, \theta_E) - \sqrt{K_p^2 + K^2(\theta, \theta_E) - 2K_PK(\theta, \theta_E)} \leq K_P \leq K(\theta, \theta_E) + \sqrt{K_p^2 + K^2(\theta, \theta_E) - 2K_PK(\theta, \theta_E)} \equiv \tilde{K}$$  \hspace{1cm} (22)

Note that the upper limit $\tilde{K}$ is such that:

$$\tilde{K} \equiv K(\theta, \theta_E) + \sqrt{K_p^2 + K^2(\theta, \theta_E) - 2K_PK(\theta, \theta_E)} \leq 2K(\theta, \theta_E)$$  \hspace{1cm} (23)

For part (1) of the Proposition, we want to show that $2K(\theta, \theta_E) < K(\tilde{\theta}, \overline{\theta}) = K^*(\tilde{\theta})$. Using the definition of $K(\theta_1, \theta_2)$ and rearranging the term, one can see that this inequality if satisfied if

$$\alpha(\tilde{\theta} - 2\theta) \geq \tilde{\alpha}(1 - d)((\tilde{\theta} - 2\theta_E)$$

which is always satisfied, since $\alpha \geq \tilde{\alpha}$ and $\theta < \theta_E$. Therefore, $K_P$ is always strictly lower than $K^*(\tilde{\theta})$.

The same logic can be applied to look at whether the type $\tilde{\theta}$ wants to deviate:

$$V(\overline{\theta}, \theta_E, K_P) \geq V(\overline{\theta}, \overline{\theta}, K(\overline{\theta}, \overline{\theta}))$$  \hspace{1cm} (24)

which gives the condition

$$K_p^2 + K^2(\overline{\theta}, \overline{\theta}) - 2K_PK(\theta, \theta_E) \leq 0$$

The solution to this inequality is

$$K \equiv K(\overline{\theta}, \theta_E) - \sqrt{K_p^2 + K^2(\overline{\theta}, \theta_E) - 2K_PK(\theta, \theta_E)} \leq K_P \leq K(\overline{\theta}, \theta_E) + \sqrt{K_p^2 + K^2(\overline{\theta}, \theta_E) - 2K_PK(\theta, \theta_E)} \equiv \tilde{K}$$  \hspace{1cm} (25)

Therefore the minimum $\tilde{K}$ is such that:

$$\tilde{K} \equiv K(\overline{\theta}, \theta_E) + \sqrt{K_p^2 + K^2(\overline{\theta}, \theta_E) - 2K_PK(\theta, \theta_E)} > 2K(\overline{\theta}, \theta_E)$$
We want to show that \(2K(\theta, \theta_E) > K(\theta, \theta) = K^*(\theta)\). Using the definition of \(K(\theta_1, \theta_2)\) and rearranging the term, this inequality is satisfied if
\[
\alpha(2\theta - \theta) \geq \bar{\alpha}(1 - d)((2\theta_E - \bar{\theta})
\]
which is always satisfied.

For part (2) of the Proposition, we look again at inequality (23) and ask under what conditions \(2K(\theta, \theta_E) \leq K(\theta_E, \theta_E)\). Using the definition of \(K(\theta_1, \theta_2)\), this inequality is satisfied if and only if
\[
\frac{2\gamma[\alpha\theta - (1 - d)\theta_E\bar{\alpha}]}{2c} \leq \frac{\gamma[\alpha\theta_E - (1 - d)\bar{\alpha}\theta_E]}{2c}
\]
which becomes
\[
\alpha(2\theta - \theta_E) \leq (1 - d)\theta_E\bar{\alpha}
\]
which is satisfied if \(\theta \leq \frac{1}{2}(1 - d)\bar{\alpha}\theta_E + \alpha\theta_E\) or \(\bar{\alpha} \geq \frac{\alpha(2\theta - \theta_E)}{1 - d}\theta_E\).

Q.E.D.

A4. Proof of Lemma 5

Note that, from inequality (23) and using the definition of \(K(\theta_1, \theta_2)\),
\[
K - K^*(\theta_E) = \frac{\gamma\alpha(\theta - \theta_E)}{2c} - \sqrt{K^2(\theta, \theta) - K^2(\theta, \theta_E)}.
\]
It is easy to verify that when \(\bar{\alpha}\) increases or \((1 - d)\) decreases, the right hand side decreases.

Q.E.D.
References


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Figure 1: The choice of restructuring $K^s$ for each $\theta$ in a separating equilibrium, relative to the second-best choice $K^* (\theta)$.