AN OPTIMAL-CONTROL BASED INTEGRATED MODEL OF SUPPLY CHAIN SCHEDULING

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Problems of supply chain scheduling are challenged by high complexity, combination of continuous and discrete processes, integrated production and transportation operations as well as dynamics and resulting requirements for adaptability and stability analysis. A possibility to address the above-named issues opens modern control theory and optimal program control in particular. Based on a combination of fundamental results of modern optimal program control theory and operations research, an original approach to supply chain scheduling is developed in order to answer the challenges of complexity, dynamics, uncertainty, and adaptivity. Supply chain schedule generation is represented as an optimal program control problem in combination with mathematical programming and interpreted as a dynamic process of operations control within an adaptive framework. The calculation procedure is based on applying Pontryagin’s maximum principle and the resulting essential reduction of problem dimensionality that is under solution at each instant of time. With the developed model, important categories of supply chain analysis such as stability and adaptability can be taken into consideration. Besides, the dimensionality of operations research-based problems can be relieved with the help of distributing model elements between an operations research (static aspects) and a control (dynamic aspects) model. In addition, operations control and flow control models are integrated and applicable for both discrete and continuous processes.

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Introduction

The term “supply chain management” (SCM) was coined in the 1980–90s. Presently, SCM is considered as the most popular strategy for improving organizational competitiveness along the entire value chain in the twenty-first century (Simchi-Levi et al. 2003; Christopher 2005; Crandall et al. 2009).

A supply chain (SC) is a network of organizations, flows, and processes wherein a number of various enterprises (suppliers, manufacturers, distributors, and retailers) collaborate (cooperate and coordinate) along the entire value chain to acquire raw materials, to convert these raw materials into specified final products, and to deliver these final products to customers (Ivanov and Sokolov 2010a).

SCM’s impact on the business performance can be estimated as up to 30%. From decisions on the SC configuration arise up to 80% of the total SC costs and up to 75% of the operational costs in SCs (Harrison et al. 2005). Regarding the merit and performance of SCM, the following figures can be shown as examples. The increase in sales and reduction in costs in the value-adding chain due to SCM amounts to 15 to 30%. Partial effects such as inventory reductions, an increase in service level, SC reliability and flexibility, a reduction in transaction costs etc. amount to 10 to 60%. These effects occur due to coordinating and harmonizing supply and demand along the entire value-adding chain to ensure iterative balances of production and logistics processes subject to full customer satisfaction.

SCs are subject to the dynamics and uncertainty of an actual execution environment. An SC schedule may be subject to numerous (about 1 million cases a year in an international large-scale supply chain) unplanned changes/disruptions and, therefore, need continuous adaptation. SC managers spend about 40–60% of their working time handling disruptions. Recent studies indicated that the scheduling needs to be considered with regard to dynamic aspects, a real performance and perturbed execution environment to fill the gap between theory and practice of scheduling (Vieira et al. 2003, Krajewski et al. 2005, Chauhan et al. 2007). Modern developments in information technologies such as RFID (Radio Frequency Identification), SCEM (Supply Chain Event Management) and mobile business provide a constructive basis to facilitate feedbacks between SC scheduling and execution control. In this setting, the extensive development of approaches and models to SC scheduling and control under the attracting adaptation methods is becoming timely and crucial.

Conventionally, scheduling deals with distribution of a given job set to the given set of resources subject to optimizing certain goal criteria (e.g., the lead-time) (Pinedo 2008). Problems of supply chain scheduling are
challenged by high complexity, combination of continuous and discrete processes, integrated production and transportation operations (Kreipl and Pinedo 2004, Ivanov and Sokolov 2010b). Besides, after long-lasting research on SC optimality from the service level’s and costs’ points of view, the research community has begun to shift to a paradigm that the performance of SCs is to consider adaptable, stable, and crisis-resistant processes to compete in a real perturbed execution environment (Sheffy 2005, Son and Venkateswaran 2007, Ivanov and Sokolov 2010a,b).

In the scheduling of SCs, a number of particular features should be taken into account. SC execution is accompanied by perturbation impacts. It requires establishing feedbacks and SC adaptation to the current execution environment (Ivanov 2010). The real dynamics, feedbacks and not determined considerations of future make SC processes non-stationary and non-linear. In addition, SC models often have high dimensionality. There are no strict criteria of decision-making and no a priori information about many SC parameters. Unlike the automatic systems, adjustment control decisions in SCs are of discrete nature as taken by managers and not by automatics. In addition, SCs may consist of both continuous (e.g., chemical production) and discrete (e.g., transportation) processes. Hence, SC scheduling is more complex than it appears in some studies.

In order to conduct research on such complex problems, a combination of different methods becomes necessary. A possibility to address the above-named issues opens modern control theory (MCT) and optimal program control (OPC) in particular. In this study, based on a combination of fundamental results of modern OPC theory and operations research (OR), an original approach to SC scheduling is developed in order to answer the challenges of complexity, dynamics, uncertainty, and adaptivity. The rest of this paper is organized as follows. In Section 1, we analyze the applicability of MCT to SC scheduling. Section 2 describes the methodical basics of the proposed approach. In Section 3 a problem statement, a conceptual model, an adaptation framework, and an optimal control algorithm for dynamic SC scheduling are presented. In Section 4, the experimental environment is discussed. We conclude the paper by summarizing the main findings and discussing future research.

1. Challenges and advantages of control theory for supply chain scheduling

In OR, optimization-based improvements in SC scheduling are usually algorithmic and based on the mathematical programming (MP) or heuristics. For the last decade, considerable advancements have been achieved in this area (Kreipl and Pinedo 2004). In recent years, the research into SCM has also been broadened to cover the whole of SC
dynamics. Along with the schedule generation, rescheduling as the process of updating an existing schedule in response to disruptions or other changes has been an influential research topic Vieira et al. (2003) underline that viewing rescheduling as a dynamic process can potentially provide a system-level perspective for real complex tasks.

With regard to OR, the following shortcomings of SC planning and scheduling can be revealed from the dynamics point of view. First, problems of high dimensionality are either reduced dimensionally or heuristics are applied. Second, dynamics of a real SC execution cannot be reflected in single-class models. Third, models of planning and execution are not explicitly interconnected in terms of uncertainty and adaptation.

In these settings, modern CT is becoming of even greater interest to researchers and practitioners in SC domain (Daganzo, 2004, Disney et al. 2006, Hoberg et al. 2007, Puigjaner and Lainez 2008, Sarimveis et al. 2008, Ivanov et al. 2010, Ivanov and Sokolov 2010b).

Regarding planning and scheduling in production and logistics, the optimal control (OC) method has been extensively developed. Optimal CT has been successfully applied for optimal planning and scheduling of continuous and discrete production processes (Kogan and Khmelnitsky 2000, Sethi and Thompson, 2006). The optimal control approach can be divided into deterministic (optimal program control — OPC) and stochastic optimal control (Fleming and Rishel, 1975). An advantage of OC is that it can be applied both for continuous and discrete systems.

OPC approach was developed by Russian mathematicians among whom the central character was Lev Semenovich Pontryagin. The importance of the work by Pontryagin et al. (1964) lies not only in a rigorous formulation of a calculus of variations problem with constrained control variables, but also in the proof of the maximum principle for optimal control problems. The economic interpretation of optimal CT and the maximum principle regarding production and inventory control (Hwang et al., 1969) have been emphasized right from the start. Feichtinger and Hartl (1985), Gaimon (1988), Khmelnitsky et al. (1997), Sethi and Thompson (2006), and Ivanov and Sokolov (2010a) provided the application of OC to production, logistics and SCM.

The first strong contribution of modern CT to the SC scheduling domain is the interpretation of scheduling not as discrete operations but as a continuous adaptive process. Hence, SC execution can be interpreted as a dynamic process of operations’ execution. Second, the possibility of covering the SC dynamics at the process level and the changes in SC and environment is also a strong contribution of CT. Third, CT allows the consideration of goal-oriented formation of SC structures and the solution
of problems in this system as a whole. Disney et al. (2006) showed that the control theoretic approach is equivalent to modeling SCs.

Therefore, important categories of SC analysis such as stability and adaptability can be taken into consideration. In addition, the dimensionality of OR-based problems can be relieved with the help of distributing model elements between an OR-based (static aspects) and a CT-based (dynamic aspects) model.

However, the CT application also has its challenges and limitations. First, the decision-making in business systems is of a discrete nature. In technical control systems, it is assumed that the control $u(t)$ can be selected continuously in time. Second, the mathematics of OPC also has its limitations. In the 1960–1970s, significant advances were made in OC techniques (Pontryagin et al. 1964, Lee and Markus 1967, Moiseev 1974). However, one of the main problems for the scheduling domain was caused by step functions and the arising sectionally continuous functions in the problems of dynamic resource distribution (Moiseev 1974).

SC tuning by means of a controller, unlike in automatic systems, occurs not within milliseconds but with a delay between the deviation occurrence and decision making depending on the disruption/change character and SC coordination. Moreover, people do not strive for a 100% guarantee of the result; they consciously tend to take risks and have different individual risk perceptions resulting in different treatment of multi-criteria problems.

2. Research Methodology

The proposed approach is based on fundamental scientific results of modern optimal CT in combination with the optimization methods of OR gained in the works by Kalinin and Sokolov (1985, 1987) regarding control theoretic interpretation of scheduling problems. Although CT and OR appear to differ in targets, presumptions, application areas, enabling technologies, and research methodologies, each compliments the others and endeavours to improve decision-making on the optimization principles (Tabak and Kuo 1971, Feichtinger and Hartl 1985, Ivanov and Sokolov 2010).

As an introduction to the methodical principles applied in this study, let us analyse the approaches to capture dynamics in SC scheduling. Without doubt, the MP approaches extensively consider the time factor within dynamic programming, mixed integer programming (MIP), stochastic, and Markov decision models. The application of the MP techniques for dynamic SC scheduling can be very successful in many cases; however, only under certain premises such as limited problem
dimensionality, unchanged objective functions (planning goals), and ended planning and scheduling horizons.

Dynamic models exist both in MP and CT but differ with regard to dependence vs. independence from the time factor. For example, the MIP models allow time to be represented as a continuous variable. However, this variable is explicitly defined by a decision-maker and influences a problem dimension. This is also the case in techniques where rescheduling occurs throughout a planning horizon. E.g., in periodic and hybrid strategies in predictive-reactive scheduling, decomposing the overall scheduling problem into smaller and static scheduling problems — if fine time granularity is required over a longer scheduling horizon — will cause the rapid increase in the number of intervals and the problem dimension may become unmanageable. Besides, the following execution stage along with the adaptation of schedules and plans usually remain beyond the MIP and other MP models. To cover the SC execution, other models (e.g., simulation) are constructed. However, the coordination and coherency of MP and simulation models require special efforts and would not necessarily be successful because of human subjectivism.

Modern CT, especially OPC, lets us also represent the scheduling problems with continuous times. However, in OPC, time becomes an independent variable which exists in a model implicitly and does not influence problem dimensionality. In addition, applying OPC for SC scheduling has another great advantage with regard to execution dynamics. CT allows supply chain dynamics to be considered both from a narrow perspective (operations dynamics within a schedule in accordance to a given plan) and a wide perspective (execution dynamics and adaptation of both schedules and plans). Hence, the problems and models of planning, scheduling, and adaptation can be integrated on unified methodical principles. The coordination and coherency of planning and execution models do not require special efforts and occur in a natural way within the united mathematical axiomatic of control theory.

In practice, static and dynamic models as well as problems with discrete and continuous times are tightly interconnected. Hence, they should not contradict each other, but rather mutually enrich themselves. An example of such model coordination is the combination of an OPC dynamic model, which represents the supply chain dynamics, and MP models, which are used at the calculation stage at each point of time with regard to dimensionally reduced allocation problems.

The first main idea of the approach proposed in this paper is to use fundamental results gained in the OPC theory for the SC scheduling domain. Lee and Markus (1967) and Moiseev (1974) proved that all conditions of optimal control existence for linear non-stationary finite-
dimensional controlled differential systems with the convex area of admissible control are valid. Consequently, if the scheduling model can be represented in the form of such a system, the application of the optimal control theory for the SC schedule generation (and not only for feedback control!) becomes possible. Therefore, the problems and models of planning, scheduling, and adaptation can be consistently integrated on a unified mathematical axiomatic of modern CT.

In the approach proposed in this paper, the consideration of dynamic flow models with dynamic resource structure models within the OPC axiomatic and on the basis of combining MP and OPC becomes possible. A peculiarity of OPC models are constraints (economic, technological, cooperation, etc.) on the control, state variables, and their combinations. The constraints and their formulation are the key to the building an OPC model.

A particular feature of the proposed approach is that the process control model will be presented as a non-stationary dynamic linear system while the non-linearity will be transferred to the model constraints. This allows us to ensure convexity and to use interval constraints. As such, the constructive possibility of discrete problem solving in a continuous manner occurs. Besides this, the required consistency between OPC and linear programming (LP) / integer programming (IP) models is ensured — although the solver works in the space of piecewise continuous functions, the control actions can be presented in the discrete form as in LP/IP models.

This is the essential structural property of the proposed approach which allows applying methods of discrete optimization for optimal control calculation. Along with the simplifications in the calculation procedure, this makes it possible to solve the assignment problem and the flow distribution problem both in discrete and continuous manner. In this aspect, the proposed approach differs from the scheduling with the help of maximum principle with only continuous control variables or discrete maximum principle which is subject to many calculation restrictions.

In the proposed dynamic scheduling model, a multi-step procedure for SC planning and scheduling is implemented. At each instant of time while calculating solutions in the dynamic model with the help of the maximum principle, the LP problems to allocate jobs to resources and IP problems for (re)distributing material and time resources are solved with conventional capacitated LP/IP algorithms.

The second main idea of the proposed approach is the calculation procedure. The works by Lee and Markus (1967) and Moiseev (1974) proved the existence of optimal control, but then a method is needed to find this control. The calculation procedure is based on the application of
Pontryagin’s maximum principle in the form of Krylov-Chernousko method (Kalinin and Sokolov 1985, 1987). The modelling procedure is based on an essential reduction of problem dimensionality that is under solution at each instant of time due to connectivity decreases. The problem dimensionality is determined by the number of independent paths in a network diagram of SC operations and by current economic, technical, and technological constraints. In its turn, the degree of algorithmic connectivity depends on the dimensionality of the main and the conjugate state vectors at the point of the solving process interruption. If the vectors are known, then the schedule calculation may be resumed after the removal of the “inactive” constraints. In contrast, MP scheduling techniques work with almost the complete list of all the operations and constraints.

The proposed model and algorithm in terms of optimal CT represents a program control of an operations complex; hence, the program control of supply chain operations execution is at the same time the supply chain schedule. On the basis of the Pontryagin’s maximum principle and corresponding optimization algorithms, the original problem of optimal control is transformed to the boundary problem. The maximum principle permits the decoupling of the dynamic problem over time using what are known as adjoint variables or shadow prices into a series of problems each of which holds at a single instant of time (Pontryagin et al. 1964). This property of optimal control is very helpful when interconnecting MP and OPC elements.

The optimal solution of the instantaneous problems can be shown to give the optimal solution to the overall problem (Pontryagin et al. 1964, 1983, Sethi and Thompson 2006). In the decomposition approach, the OPC is similar to the rolling scheduling. However, as in OPC, time is an independent variable which exists in a model implicitly and does not influence problem dimensionality, the decomposing of the scheduling horizon with the help of the maximum principle also does not influence the problem dimensionality.

The proposed approach is also similar to the predictive-reactive rescheduling regarding interlinking the stages of schedule generation and update (Vieira et al. 2003, van de Vonder et al. 2007). At the same time, the proposed approach reaches beyond the traditional predictive-reactive rescheduling and incorporates the elements of control theoretic approach to dynamic scheduling as highlighted by Vieira et al. (2003).

3. Scheduling model and algorithm

We consider an SC that is planned and scheduled from the point of view of the SC optimization as a whole. The total SC performance is the main premise of the SC planning and scheduling. Such cases are very
common in SCs with a possibility for centralized scheduling, e.g., with strong original equipment manufacturers (OEM), by applying the CPFR (collaborative planning, forecasting and replenishment) coordination strategy or in SCs which are managed by logistics service providers (3PL/4PL). Besides, this case reflects the ideology of SCM — the optimization of an SC on the whole and not its parts. Even if the decentralization and antagonistic goals of enterprises in an SC do not allow the implementation of the general optimal schedule, this solution may be considered as an orientation for schedule changes with regard to local enterprise goals.

The goal is to (re)plan and (re)schedule the customer orders in SC functioning subject to the maximization of the service level (the general volume of fulfilled orders in accordance with the delivery plan), the minimization of penalties for breaking delivery terms, and the minimization of inequality in resource usage rate in the SC (the requirement for SC collaboration).

Let us introduce the set theory-based problem statement. Let \( \mathcal{B} = \{ B^i \}, i \in \mathcal{N} \), \( \mathcal{N} = \{1, \ldots, n\} \) be a set of customers’ orders that can be realized in a SC. Each order is characterized by operations \( D^i_\mu \), \( \mu = 1, \ldots, s ; i = 1, \ldots, n \). Let \( \mathcal{C} = \{ C^j \}, j \in M \) be a set of enterprises in a SC. The jobs’ realization with these resources is connected to the flows \( P^{(i,j)} = \{ p^{(i,j)}_{\rho} \}, \rho = 1, \ldots, \rho; \mu = 1, \ldots, s \), where \( \rho = 1, \ldots, \rho \) is the enumeration of flows.

The blocking of certain arcs in the dynamic graph resulting from the above-given set-theoretic SC structure description (Ivanov et al. 2010) is used to reflect non-cooperative relations among certain enterprises in a SC. The duration of operations and the resource productivity along with lower and upper bounds of perturbation impacts on resource availability and productivity are known for a given scheduling horizon. Enterprises can block the availability of their resources for certain periods of time (in the form of time-spatial constraints).

This problem statement is static. The dynamics formal statement of the scheduling problem will be produced, as it was noted above, via a dynamic interpretation of the operations’ execution processes.

The detailed description of dynamic SC scheduling model is presented by Ivanov and Sokolov (2010b). Let us consider the aggregated form of this model. The planning and scheduling problem can be formulated as the following problem of dynamic system optimal program control. This is necessary to find an allowable control \( u(t), t \in [T_0, T_f] \) that ensures for the dynamic models of operations, resource and flow control meeting the requirements \( q^{(1)}(x,u) = 0, q^{(2)}(x,u) \leq 0 \) for a set of economic and
technological constraints and guides the dynamic system \( \dot{x} = f(x, u, t) \) from the initial state \( x_0 \) to the specified final state \( x_f \). If there are several allowable controls (schedules), then the best one (optimal) should be selected in order to maximize (minimize) \( J_c \). In terms of OPC, the program control of SC operations execution is at the same time the SC schedule.

According to Lee and Markus (1967), along with the initial class \( K \) formed via constraints \( q^{(1)} \) and \( q^{(2)} \) describing the domain \( Q(x(t)) \), an extended class \( \tilde{K} \) of control inputs can be considered. In the extended class \( \tilde{K} \) the relay constraints \( u_{ij}^{(o)}(t) \in [0,1] \) are substituted for a less strict one \( u_{ij}^{(o)}(t) \in [0,1] \) (u is substituted for \( \tilde{u} \)). In this case, an extended domain \( \tilde{Q}(x(t)) \) of allowable control inputs may be formed through special transformations ensuring the convexity and compactness of \( Q(x(t)) \) (Lee and Markus 1967, Moiseev 1974, Kalinin and Sokolov 1985, 1987).

An analysis of Lee and Markus (1967) and Okhtilev et al. (2006) confirms that all the conditions of optimal control existence for the extended control class \( \tilde{K} \) are valid. The work by Okhtilev et al. (2006) has shown that, if in a given class of permissible control actions \( \tilde{K} \), the optimal control \( \tilde{u}(t) \) exists, then, as arises from the local section method, the control \( \tilde{u}(t) \) returns at each instant of time \( t \in (T_0, T_1) \) at the set \( \tilde{Q}(x(t)) \) a maximum to the following Hamiltonian \{xe "Hamiltonian"\} (1)–(5)

\[
H(x^*(t), u^*(t), y^*(t)) = \max_{u(t) \in \tilde{Q}(x(t))} \sum_{t=1}^{4} H_i(x(t), u(t), y(t)), \tag{1}
\]

\[
H_1 = \sum_{i=1}^{n} \sum_{\mu=1}^{n} \sum_{j=1}^{n} \left[ \psi_{ij}^{(o)} \cdot \epsilon_{ij} + \psi_{ij}^{(k)} + \lambda_2 \alpha_{ij}^{(o)} \right] u_{ij}^{(o)}, \tag{2}
\]

\[
H_2 = \sum_{i=1}^{n} \sum_{\mu=1}^{n} \sum_{j=1}^{n} \sum_{\eta=1}^{n} \psi_{ij}^{(o)} \psi_{ij}^{(k)} \psi_{ij}^{(o)} \psi_{ij}^{(o)} u_{ij}^{(o)}, \tag{3}
\]

\[
H_3 = \sum_{i=1}^{n} \sum_{\mu=1}^{n} \sum_{j=1}^{n} \left[ \psi_{ij}^{(f)} + \lambda_6 \beta_{ij}^{(f)} \right] u_{ij}^{(f)}, \tag{4}
\]

\[
H_4 = \sum_{i=1}^{n} \sum_{\mu=1}^{n} \sum_{j=1}^{n} \sum_{\eta=1}^{n} \psi_{ij}^{(f)} u_{ij}^{(f)}, \tag{5}
\]

where \( y(t) \) is the vector of the conjunctive equation system at each instant of time.

Maximization of the Hamiltonian \( H_i \) at each instant of time along with considering constraints dynamic model comes to solving the assignment problem. Maximization of the Hamiltonian \( H_2 \) at each instant of time along with considering constraints of the dynamic model comes to solving the transportation problem or linear programming problem. Maximization of the Hamiltonians \( H_3 \) and \( H_4 \) at each instant of time along with considering
constraints of the dynamic model comes to solving the linear programming problem. Here, the conjugate system and the transversality conditions can be written as presented in (Ivanov and Sokolov 2010) based on the works by Pontryagin et al. (1964), Moiseev (1974), Kalinin and Sokolov (1985) and (1987).

Equations (2) and (3) are a discrete form of an allocation problem at each moment \( t, \ t \in (T_0, T_f] \). Equations (4) and (5) can be interpreted as the linear programming problem. The reduction of problem dimensionality at each instant of time in the calculation process is ensured due to the description of recurrent operations. At each instant of time, only those operations are considered that meet the requirements of constraints (the so-called active operations). This means that the scheduling problem at each point of time is essentially reduced and MP techniques can be applied.

The Hamiltonians (2) and (3) can be maximized when the constraints of the dynamic model satisfy for the corresponding variables \( u_{ij}^{(o)} \) and \( u_{jnp}^{(o)} \). In this case, only a part of the constraints in the dynamic model will be considered while solving current allocation problems with MP techniques. Even this fact allows us to assert that the current dimensionality of allocation problems at each point of time is significantly smaller than those in conventional MP formulations. So there is no need to drag all the constraints through the scheduling period.

Thus the problem dimensionality depends on the amount of active operations only. Therefore the relaxed problem can be solved instead of the initial one to receive an optimal allowable control. It was shown by Kalinin and Sokolov (1985, 1987) that the stated necessary conditions of optimality are also sufficient. Hence, a scheduling problem for SCs can be reduced to a boundary problem with the help of the local section method.

Let us consider the algorithm for the indicated boundary problem. In this case, a “first approach” for launching the optimization procedure is required. This can be, e.g., a heuristics schedule \( \pi(t) \) that can be generated either by a simple priority rule (e.g., first-in-first-out — FIFO) or by a high-level heuristic such as a genetic algorithm (or any other heuristics that are widely applied for supply chain scheduling). Then, the scheme of computation can be stated as follows:

**Step 1** An initial solution \( \bar{\pi}(t), t \in (T_0, T_f] \) (an arbitrary allowable control, in other words, allowable schedule) is calculated with the help of a heuristic and \( r = 0 \).

**Step 2** The parameters of the gained schedule \( \pi(t), t \in (T_0, T_f] \) are put into the dynamic model and integrated. As a result of the dynamic model...
run, a new trajectory of operation states \( x^{(r)}(t) \) is received. Besides, if \( t = T_f \) then the record value \( J_G^{(r)} = J_G^{(r)} \) can be calculated.

**Step 3** Then, the transversality conditions are evaluated. The conjugate system is integrated subject to \( u(t) = \bar{u}(t) \) and over the interval from \( t = T_f \) to \( t = T_0 \). The concrete forms of the conjugate system and the transversality conditions are presented by Ivanov and Sokolov (2010a, pp.197–198). For time \( t = T_0 \), the first approximation \( \psi^{(r)}(T_0) \) is received as a result. Here, the iteration number \( r = 0 \) is completed.

**Step 4** From the point \( t = T_0 \) onwards, the control \( u^{(r+1)}(t) \) is determined \((r = 0,1,2,...\) is the number of iterations) through conjunctive system. Along with maximization of the Hamiltonian, the main system of equations and the conjugate one are integrated. The maximization is performed with regard to MP problems of (2)–(5) under the presence of constraints of the dynamic model (Ivanov and Sokolov 2010b) at each time point.

The iterative process of the optimal schedule search is terminated under the following circumstances: either the allowable solution to the problem is determined during the solving of a relaxed problem, or at the fourth step of the algorithm after the integration we receive:

\[
\begin{align*}
|J_G^{(r+1)} - J_G^{(r)}| &< \epsilon_1, \\
\|u^{(r+1)} - u^{(r)}\| &< \epsilon_2,
\end{align*}
\]

(6)

where \( \epsilon_1, \epsilon_2 \) are given small values, \( r = 0,1,2,... \). If condition (6) is not satisfied, then the third step is repeated, etc.

4. **Computational procedure and discussion of results**

For the experiments, we elaborated the model in a software package. The software has three modes of operation with regard to scheduling and an additional mode to analyse stability of the schedules. This mode is beyond the scope of this paper.

The first mode includes the interactive generation/preparation of the input data. The second mode lies in the evaluation of heuristic and optimal SC schedules. The third mode provides interactive selection and visualization of SC schedule and report generation. An end user can select the modes of program run, set and display data via a hierarchical menu.

The first step is the input data generation. These data create SC structure and the environment on which scheduling will be performed. These data refer to the problem statement and models of Section 3 in this paper. The data can also be input by a user. After setting up SC structures, planning goals and environment parameters (customer orders and possible uncertainty impacts), the scheduling algorithm is then run. The algorithm of dynamic control is programmed by us; for the optimization of problems
(1)–(4) under the presence of constraints of the dynamic model at each time point by means of MP techniques, the OPC algorithm addresses the MP library of the MS Excel Solver.

The schedule can be analysed with regard to performance indicators. Subsequently, parameters of the SC structures and the environment can be tuned if the decision-maker is not satisfied with the values of performance indicators.

Of course, these 15 parameters should not be tuned all at once. The tuning depends a great deal on the SC strategy. In the case of a responsive strategy, the increase in the amount of resources and capacities leads in the direction to improving the values of service level and to increasing the amount and volumes of customers’ orders. In the case of an efficient strategy, resource consumption and penalties should be reduced as much as possible even if the lead times and supply cycles would increase and the service level decrease. With regard to perturbation impacts, an SC planner can also analyse different alternative SC plans, fill these plans with reliability and flexibility elements to different extents, and then analyse how these changes influence the key performance indicators. In the current version of the software package, this tuning is still performed manually; hence we are still unable to provide either justified conclusions of recommended settings of parameters or established methods for tuning. However, the extension of the software prototype in this direction is under development. The conducted experiments showed that the application of the presented dynamic scheduling model is especially useful for the problems where a number of operations are arranged in a certain order (e.g., technological restrictions). This is the case in SC planning and scheduling.

The building of the scheduling model within the proved theorems and axioms of the optimal CT (Lee and Markus 1967) allows us to consider the found solutions as optimal (see the proofs of the maximum principle in Pontryagin et al. (1964) and the application of maximum principle to economic problems by Sethi and Thompson (2006). Based on the optimal solutions, we can also methodically justify the quality of different heuristics that have launched the optimization procedure.

**Conclusions**

In this paper, we addressed the challenges and perspectives that delineate dynamics in SC scheduling, commented on methodical issues, and described one specific context, model and algorithm in the dynamic SC scheduling area. For the stage of SC schedule generation, we formulated the scheduling model as a linear non-stationary finite-dimensional controlled differential system with the convex area of
admissible control and a reconfigurable structure. As, according to the theorems of CT fundamentals (Lee and Markus 1967, Moiseev 1974), the optimal control exists and can be calculated (Pontryagin et al. 1964) for this model class, the results of CT can also be used for the SC scheduling domain both for SC schedule generation and adaptive feedback control. Therefore, the problems and models of planning, scheduling, and adaptation can be consistently integrated on a unified mathematical axiomatic of modern CT. In addition, in the proposed dynamic scheduling model, operations control (the assignment problem) and flow control (the transportation or linear programming problem) models are integrated and applicable for both discrete and continuous processes.

The proposed model has some specific features in comparison with classic optimal control problems. The first feature is that the right parts of the differential equations undergo discontinuity at the beginning of interaction zones. The considered problems can be regarded as control problems with intermediate conditions. The second feature is the multi-criteria nature of the problems. The third feature is concerned with the influence of uncertainty factors. The fourth feature is the form of time-spatial, technical, and technological non-linear conditions that are mainly considered in control constraints and boundary conditions.

In distributing static and dynamic elements of the considered problem between a static LP model (for aggregate planning within certain time intervals of structural constancy) and a dynamic OPC model (for representing SC structure dynamics and planning time-dependent processes within the intervals of structural constancy), it became possible to state a solvable integrated problem instead of partial problem (i.e., production scheduling, transportation scheduling, and execution control) that have been previously treated inconsistently and in isolation. Such a problem formulation is near to the real-world settings.

Finally, let us discuss the limitations of the proposed approach and future work. We considered an SC that is planned and scheduled from the point of view of the SC optimization as a whole. The total SC performance is, in this case, the main premise of SC planning and scheduling. However, there are many other SC management strategies with decentralized planning, schedule coordination and performance evaluation. These cases remain beyond the scope of this paper.

For practical problems, the usage of the FIFO/LIFO algorithms for the launching of the optimization algorithm is obviously not the best way as the convergence speed of the optimization algorithm depends greatly on the quality of this “first” approach. Hence, high-level heuristics should be applied. This aspect will be investigated in our future research.
In addition, the following future research needs can be indicated. For the SCs with decentralized planning and performance evaluation modes, we intend to apply the differential games as well as coalition and cooperative games. The advantage is that the methodical basics of these tools are practically identical to the optimal control model presented in this paper. Besides, a further extension of methodical basics towards a combination with agent-based modelling is included in our future research plans.

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